Informations for Contributors

Contributors should follow as closely as possible the rules below:

Manuscripts should be typed (double-spaced) in Prestige-Elite characters (IBM-type), on one side of plain paper 21 cm x 29.7 cm, with a 2 cm margin on the left and right hand sides as well as on the bottom, and with a 3 cm margin at the top (as indicated by the frame drawn on this page).

Title of paper. Titles should be carefully worded to include only key words.

Abstract. The abstract of a paper should be informative rather than descriptive. It is not a table of contents. The abstract should be suitable for separate publication and should include all words useful for indexing. Its length should be limited to one typescript page.

Table of contents. Long papers may include a table of contents following the abstract.

Footnotes. Because footnotes are distracting, they should be avoided as much as possible.

Mathematics. For papers with complicated notation, a list of symbols and their definitions should be provided as an appendix. All characters that are available on standard typewriters should be typed in equations as well as text. Symbols that must be handwritten should be identified by notes in the margin. Angle brackets should be allowed around equations so that type can be marked for the printer. Where an accent or underscore has been used to designate a special type face (e.g., boldface for vectors, script for transforms, sans serif for tensors), the type should be specified by a note in the margin. Bars cannot be set over superscripts or extended over more than one character. Therefore angle brackets are preferable to overbars to denote averages, and superscript symbols (such as \( \mathbb{R} \), and \( \mathcal{M} \)) are preferable to accents over characters. Care should be taken to distinguish between the letter O and zero, the letter I and the number one, kappa and k, \( \mu \) and the letter \( \mu \), nu and \( \nu \), eta and \( \eta \), also subscripts and superscripts should be clearly noted and easily distinguished. Unusual symbols should be avoided.

Acknowledgments. Only significant contributions by professional colleagues, financial support, or institutional sponsorship should be included in acknowledgments.

References. A complete and accurate list of references is of major importance in review papers. All listed references should be cited in text. A complete reference to a periodical gives: author(s), title of article, name of journal, volume number, initial and final page numbers (or statement "in press"), and year published. A reference to an article in a book, pages cited, publisher, publisher's location, and year published. When a paper presented at a meeting is referenced, the location, date, and sponsor of the meeting should be given. References to foreign works should indicate whether the original or a translation is cited. Unpublished communications can be referred to in text but should not be listed. Page numbers should be included in reference citations following direct quotations in text. If the same information has been published in more than one place, give the most accessible reference; e.g., a textbook is preferable to a journal; a journal is preferable to a technical report.

Tables. Tables are numbered serially with Arabic numerals, in the order of their citation in text. Each table should have a title, and each column, including the first, should have a heading. Column headings should be arranged to that their relation to the data is clear.

Footnotes for the tables should appear below the final double rule and should be indicated by a, b, c, etc. Each table should be referred to in the text.

Illustrations. Original drawings of sharply focused glossy prints should be supplied, with two clear Xerox copies of each for the reviewers. Maximum size for figure copy is (25.4 x 40.6 cm). After reduction to printed page size, the smallest lettering or symbol on a figure should not be less than 0.1 cm high; the largest should not exceed 0.3 cm. All figures should be cited in text and numbered in the order of citation. Figure legends should be submitted together on one or more sheets, not separately with the figures.

Nailing. Typescripts should be packaged in stout padded or stiff containers; figure copy should be protected with stiff cardboard.
# TABLE OF CONTENTS

**Bull. d’Ing., n° 49**

**OBITUARY**

## PART 1: INTERNAL MATTERS

- Announcement ................................................................. 6

## PART 2: CONTRIBUTING PAPERS

- Program of Special Study Group 3.37 and 3.40 Workshop ............... 9
- List of Participants .......................................................... 12
- Preamble by G. Balmino .................................................... 14
- Introduction to the Problems of High-Precision Gravimetry by E. Groten 15
- Texts of Papers Presented at the Workshop ................................ 18
- Resolutions ................................................................. 200

## PART 3

- List of Publications Dealing with Gravity Matters Received at B.G.I. (Bull. n° 48 continued) .................................................. 202
OBITUARY

Professor Anton GRAF

Professor Anton Graf, one of the pioneers of modern gravimetry, died in September 1981 at the age of 81. He was an Honorary Professor at the Technical University of Munich where he taught gravimetry until 1965; at the same time he retired as a Professor from the Munich Engineering Academy.

Prof. Graf was one of the leading gravimetrist in the world; together with Sam Worden, Lucien LaCoste and others, he created the instrumentation of relative gravimetry. His Gs-type gravimeter, which he built as a senior scientist with the Askania-Werke at Berlin, is the best known linear gravimeter. Because of its straightforward construction it could easily be modified into a modern high-precision earth-tide meter; A. Graf's sea tide gravimeter led to a breakthrough in ocean gravimetry. His earth tide vertical pendulum is still one of the most reliable and dependable instruments for measuring horizontal earth tide components. During his last years he quietly worked on some modifications of his instrumentation.

Anton Graf was an outstanding scientist of international reputation, and a sympathetic and kind human being to whom we owe a lot.

E. GROten, Darmstadt

Walter GROSSMANN
1897 - 1980

On October 13, 1980, the geodetic community lost Professor Dr.-Ing. E.H. Walter Grossmann, Hannover, Germany.

Born on the 6th April 1897 in Norden, Frisia, Grossmann studied geodesy and surveying at the Universities of Bonn and Berlin. He obtained practical surveying experiences at the Prussian Cadastral Administration, performed fruitful scientific work at the Geodetic Institute Potsdam, and assumed responsible organiza-
tional and management duties at the Ministry of Interior and the German Ordnance Survey. Appointed professor of geodesy, he acted from 1943 to 1968 as director of the Geodetic Institute, Technical University of Hannover, fascinating more than 450 students of geodesy and surveying with his profound knowledge and excellent didactic capacity, and leading more than 30 of them to the doctor degree. His well-known textbooks about surveying, adjustment computations, and ellipsoidal geodesy and conformal mapping accompanied the students of geodesy, surveying and civil engineering for more than 30 years.

Grossmann's and his Institute's research concentrated on adjustment theory, ellipsoidal calculations, instrumental developments and the establishment of the gravity network in the Federal Republic of Germany, based on new relative pendulum and gravimeter observations carried out in the 1950ies; and was appointed national representative at the International Gravimetric Commission from 1953 to 1963. Under his responsibility the Euroafrican Gravity Network between Hammerfest and Capetown was observed by his collaborators between 1958 and 1964, employing Askania GS 12 gravity meters. The results of these extensive surveys are a significant contribution to the International Gravity Standardization Net 1971 in that area. As part of the international attempts to observe the earth tide signal at different regions of the world, Grossmann established an earth tide station at Bad Grund, Harz, and started observations with gravimeters and horizontal pendulums there. He served as secretary of the I.A.G. section "Gravimetry" from 1960 to 1963, and actively participated at the I.G.C. meetings since 1953.

Grossmann's creative power was based on a marked sense of duty, clear judgement and keen self-criticism, to his students and collaborators he always gave friendly advice and promoted them by helpful discussions. The author personally feels happy and thankful to have had the chance of being a student and co-worker of Walter Grossmann. Based on Grossmann's work, gravimetric research at Hannover University still forms an important part at the Geodetic Department activities.

Wolfgang TORGE, Hannover
PART I : INTERNAL MATTERS
On the occasion of the publication in this issue of quite a large number of papers, presented at the SSG. 3.37 & 3.40 Workshop held in Paris last October, and considering the pain which our secretariat has had in assembling and modifying most of the manuscripts, I want to recall that all contributing papers should be prepared in a manner which satisfies, as closely as possible, the instructions listed in the front page. Among these specifications, the margin constraints are the most severe for the final printing and therefore must be satisfied in all cases.

Papers which will not follow these rules will be sent back to their authors.

G. BALMINO
Director
ANNOUNCEMENT

SECTION OF THE EARTH SCIENCES
PRESIDIOUM OF THE ACADEMY OF SCIENCES, USSR;
DEPARTMENT OF GEOLOGY, GEOPHYSICS AND GEOCHEMISTRY
ACADEMY OF SCIENCES, USSR;
O.Y. SHMIDT INSTITUTE OF THE PHYSICS OF THE EARTH,
ACADEMY OF SCIENCES, USSR;
SOVIET GEOPHYSICAL COMMITTEE ACADEMY OF SCIENCES, USSR

Inform, that on August 10, 1981 Prof. J.D. BOULANGER, corresponding member of Ac. Sci., USSR, celebrated his 70th anniversary and 50 year of his scientific activity and co-ordinational work, in his capacity of Vice-President of Soviet Geophysical Committee, Chairman of the Section of Geodesy, Chairman of KAPG, President of International Commission on Recent Crustal Movements, Chief of Department of Gravimetry and Geodesy.

Celebration was held on August 10, 1981; at the Institute of Physics of the Earth, B. Grouzinskaya, 10, 123810 Moscow, USSR.
PART II : CONTRIBUTING PAPERS
INTERNATIONAL ASSOCIATION OF GEODESY

SPECIAL STUDY GROUP 3.37 AND SPECIAL STUDY GROUP 3.40

Monday, October 26, 1981

9.00 a.m. Registration in the "salle bleue" of IPG.
9.30 OPENING SESSION: Introduction and Welcome
(Pr. Boulanger, Pr. Groten, Dr. Tanner, Pr. Torge (IAG representative etc.))

FIRST SESSION (Chairman: Pr. Morelli)

10.30 Faller, J.E., R.L. Rinker & M.A. Zumbeerge: The JILA absolute gravimeter
11.00-11.20 Coffee Break
11.20 Boulanger, Yu.*: Results of repeated gravity determinations by absolute gravimeter GABL

12.00 Fischer, W., H.G. Kahle & I. Marson: Absolute and relative gravity measurements in Switzerland with special emphasis on a New Swiss National Gravity Net

SECOND SESSION (Chairman: Dr. Tanner)

14.30 Kanngieser E. & W. Torge: Calibration of LaCoste-Romberg Gravity meters model G and D
15.00 Gerstenecker C.: Temperature effects on LaCoste Gravity meters
15.30 Becker M.: Results of circular error studies with LaCoste-Romberg model D meters
16.00-16.30 Coffee Break
16.30 Lambert A.: Non-linearities in LaCoste and Romberg model D gravimeters by the Cloudcroft Junior method
17.00 Boulanger Yu. and al.*: Standardization of gravimeters and gravity variation.

* Title changed.
Tuesday, October 27, 1981

THIRD SESSION (Chairman : Pr. Boulanger)

9.00  Grafarend E. : Space-Time representation of the gravity field.

9.30  Groten E. : Tidal corrections for absolute gravimetry.

10.00 Goodking J.M. : Precision of the Superconducting Gravimeters Determined by Simultaneous Measurements with Two Instruments.

10.30-11.00 Coffee break

11.00 Torge W. & E. Kamngieser Current observation of gravity variations and vertical crustal movements during the recent rifting process in Northern Iceland


Lunch time : Meeting of the recommendation committee.

FOURTH SESSION (Chairman : Dr. Balmino)

14.00 Coron S. : Recent results of gravity variations around Mount Etna.


14.45-15.00 Coffee Break

15.00-16.30 Panel discussion (Groten, Becker, Lambert, Torge) :

The D-meter campaign

a) objectives and goals ; techniques to be applied

b) detailed time schedules ; logistics ; calibration

c) details of performance (tidal corrections, reduction techniques, observation techniques, data processing)

16.30-16.45 Break

16.45-17.45 CLOSING SESSION : Adoption of recommendations

17.45 COCKTAIL OFFERED BY THE PRESIDENT OF THE COMITE NATIONAL FRANCAIS DE GEODESIE ET GEOPHYSIQUE (C.N.F.G.G.)

Wednesday, October 28, 1981

10.30 Visit to the absolute gravity station at BIPM where several absolute gravimeters will take part in the International Absolute Gravity Campaign 1981 doing comparison of the various apparatus from the Soviet Union, U.S., France and Italy.
Specialized meeting for the International Bathymetric Chart of the Mediterranean and overlay maps in gravimetry, magnetism and geology.

Attendees: Balmino, Boulanger, Groten, Moralli, Sarrailh, Sajins, Tanner, Wilcox.
LIST OF PARTICIPANTS

BALMINO, G., B.G.I., France
BECKER, M., TH Darmstadt, F.R.G.
BOEDECKER, G., D.G.F.I. München, F.R.G.
BOUCHER, C., I.G.N., France
BOULANGER, J.D., Inst. of the Earth, U.S.S.R.
BREIN, R., IFAG, F.R.G.
CAGIENARD, P.J., ETH Zürich, Switzerland
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JEUDY, L., Earth Physics Branch, Canada
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LAMBERT, A., Earth Physics Branch, Canada
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MARSON, I., Univ. Trieste, Italy
MORELLI, C., Univ. Trieste, Italy
OGIER, M., B.R.G.M., France
PETTERSSON, L., National Land Survey, Sweden
POITEVIN, C., I.G.N., Belgium
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SARRAILH, M., B.C.I., France
SPITA, W., DMAHTC, U.S.A.
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SZABO, Z., Hungarian Geophysical Institute, Hungary
TANNER, Earth Physics Branch, Canada
TORGES, W., Institute für Theor. Geodäsie, Universität Hannover, F.R.G.
WENG, L.W., Geodetic Institute, Denmark
WILCOX, L.E., DMAAC, U.S.A.
ZUMBERGE, M.A., JILA Univ. of Colorado, Boulder, U.S.A.
PREAMBLE

The workshops of SSG 3.37 and SSG 3.40 (of which the chairmen are Prof. E. Groten and Prof. J.D. Boulanger, respectively), were held simultaneously in Paris, Institut de Physique du Globe, from October 26 to 28, 1981, under the sponsorship of IAG.

This meeting, devoted to special techniques in gravimetry and long term non tidal, variations of gravity, took place at a crucial time since it coincided with the intercomparison in Sèvres (B.I.P.M.) of four absolute gravimeters, namely: the Sakuma type apparatus (BIPM), commercialized by Jaeger Company; the Academy of Sciences of URSS apparatus; the Faller instrument (JILA Univ. of Colorado, USA); the Hammond instrument (AFGL, U.S.A.).

47 persons in total participated to these manifestations, and to a few other meetings which were also held at this occasion (IAG section 3 bureau meeting, held on October 25; Mediterranean Gravity Map meeting, October 28).

In this 49 issue of the Bulletin d'Information, we present only the activities and most of the contributed papers of SSG 3.37-3.40 workshops. A detail account of the intercomparison activities at BIPM will be given in a subsequent issue when the reductions are carried out and the results made available.

G. BALMINO
INTRODUCTION TO THE PROBLEMS OF HIGH-PRECISION GRAVIMETRY

E. GROTEN

The basic aim of this workshop is a discussion of high precision techniques in gravimetry in view of (1) improved accuracy and (2) apparent and physical variations of gravity. Without going into details the complexity of the associated problems is perhaps best explained utilizing the diagrams shown in Fig. 1. It shows the amplitudes of those effects influencing gravity measurements which have periods $T$ of about $0.1 < T < 10^{10}$ seconds of time. Under favorable conditions perturbations with periods less than a few minutes in relative gravimetry and less than 1 hour in absolute gravimetry can be averaged out or eliminated. Such effects are mainly due to industrial and other artificial influences with periods $T < 10$ seconds, seismic activities with $T < 10^2$ seconds; the detection of eigenvibrations of the Earth by precise gravimetry is a goal in itself. Similarly (natural and artificial) hydrological as well as meteorological effects can be studied and, consequently, are topics of gravimetric interest; their maximum amplitudes are expected at $T \approx 1$ day and $T \approx 1$ year. Associated are temperature perturbations. The one-year period (and the 14 month period) of polar motion may have similar amplitudes as those effects. Other astronomical effects connected with the motion of the Earth in space are expected to have smaller (of the order of a few microgal) amplitudes. Secular tectonic and geological processes are often overlapped by man-made processes so that their separation is of fundamental interest. Load tides still need further investigations over the tidal spectrum from ter-diurnal to long-period constituents; the Earth's response to stationary tides, ter-diurnal tides and various other constituents is, to some extent, still an open question.

With present techniques accuracy of $\pm 10$ microgal seems to be feasible in high-precision gravimetry. With special techniques we might obtain accuracy of a few microgal by repeated observations over periods $T > 1$ week or so. However, for intervals much longer than some years there arise problems of stability. Fig. 1 reveals the fact that accuracy of the order of $\pm 1$ or $\pm 2$ microgal could lead to substantial improvement of the impact of high-precision gravimetry.

None of the aforementioned effects and perturbations can be left out in the further investigations if the efficiency of precise gravimetry should increase. Stability of relative stationary and field equipment (in terms of mechanical and
superconducting gravimeters), as well as of absolute gravimeters and their resolutions have been substantially improved during the past but still need further improvement. Further details are, e.g., found in (Elstner, 1980).

References:

Accuracies of absolute and relative gravimetry and related questions

![Graph showing tidal frequencies and their corresponding activities.]

"averaging out"

Tidal frequencies \((M_3, M_f, M_m, M_n)\)

- **a**: earthquake activities
- **b**: microseismic activities
- **c**: menmade (artificial effects) activities
- **d**: eigen-vibrations of the earth
- **e**: hydrological and meteorological effects
- **f**: secular processes
- **g**: astronomical effects (polar motion etc.)
- **h**: present accuracy
- **i**: special techniques (short term) - 1yr

Fig. 1

J.A. HAMMOND

THE JILA ABSOLUTE GRAVIMETER

J.E. FALLER, R.L. RINKER, M.A. ZUMBERGE

"Papers not sent on time"
ON NON-TIDAL GRAVITY VARIATIONS

There are many indications in world literature that repeated measurements reveal considerable gravity variations. But due to inadequate metrological facilities most of them are uncomparable. At the same time their authors identify these discrepancies with the changes in the gravity field of the Earth in time. Such experimental evaluations often differ by more than a couple of orders. Moreover, it should be noted that until quite recent times all gravity determinations were made by relative methods, which are in principle unable to reveal changes of global character.

Theoretical evaluations of possible changes, based on the analysis of different models of the Earth and the effect on them of both exogenic and endogenic factors, are also essentially different.

Therefore, the problem of gravity field variations in time is sufficiently complicated and to solve it we must develop theoretical and experimental research. Since its solution is of great importance for an extensive complex of natural sciences, each new result indicating to the change or stability of gravity in time should be carefully justified and first of all from the viewpoint of metrological facilities of measurements.

Repeated gravimetric ties at the point in Ledovo (Moscow) /1/ and repeated measurements made in 1955 and 1967 along the line Potsdam, Riga, Moscow, Kazan, Sverdlovsk, Chita, Takhtamyga, Petropavlovsk-on-Kamchatka, and also along the line Tbilisi, Ashkhabad, Balkhash, Alma-Ata, Tashkent, Dushanbe /2/, have shown that all measured differences were less that the errors of their determination, and if gravity variations exist, then they exceed ±0.02 mgal/\textit{j}.

The measurements, carried out by the Institute of Physics of the Earth, USSR Academy of Sciences, in 1958 and then in 1968 and 1974 by the International Gravimetric Expedition, organised by the Commission of the academies of Sciences of the Socialist countries.
on the complex problem "Planetary geophysical researches" (KAPG) jointly with the Geodetic Services of these countries, have demonstrated that the non-tidal changes of gravity at the points in Tallinn, Warsaw, Krakow, Prague, Budapest, Bucharest and Sofia in relation to Potsdam are small, smaller than the errors of their determinations and cannot exceed a few mcgal/\text{y}.

Beginning in 1967, Prof. A. Sakuma systematically made absolute determinations of gravity with his stationary gravimeter in Sèvres. The results of these measurements for 1968-1969 were used to determine the zero of the IGSN-71 system.

The study of non-tidal changes of gravity with the Soviet absolute gravimeter GABL were started in 1976. For this purpose many times repeated determinations at the point in Ledovo, in Novosibirsk, in 1976, 1978 and 1980 in Potsdam were carried out. The initial epoch is established for the study of gravity variations in Singapore, at five points on the Australian continent, in Tasmania, New Guinea Papua, at two points in Finland.

In 1977 in Sèvres the GABL gravimeter was compared with Prof. Sakuma's instrument and with the instrument of the G. Colonetti Institute of Metrology in Torino. These measurements had excellent correlation, but the obtained gravity value was by 48 mcgal greater than that given in the Catalogue of Points of IGSN-71 System /3/.

Similar results were received for Potsdam /±48 mcgal/, Helsinki /±35 mcgal/ and Ledovo /±49 mcgal/ (Table 2). These data are in amazing agreement. Their spread is only 11 mcgal, whereas the average value of difference ±45 mcgal was obtained with the error of ±2.6 mcgal.

Somewhat similar values, but less accurate, were obtained for the points in Australia. On that continent the average variation for 4 points was ±24.5 ± 12 mcgal (Table 3).

These deviations in measurements could have occurred only as the result of the change of zero of the IGSN-71 system in the period from 1970 to 1973-1974. According to Prof. Sakuma's measurements, at that time gravity increased by 50 mcgal.

The cause of this change is not clear. It might be the consequence of accumulation of systematic errors in the process of measurements at Sèvres, or the effect of local changes, perhaps, in hydrological conditions in the region of observations. The gravity change at Sèvres, however, is beyond doubt and reaches ±45±2.6 mcgal.
We shall now analyse the results of repeated gravity determinations at the points in Ledovo, Novosibirsk and Potsdam. Preliminary analysis of these measurements showed that at all three points gravity had identical changes. In actual fact, in the time interval from 1975.8 to 1978.0 the maximal change of gravity was observed at these points at the rate of:

\[
\begin{align*}
\text{Potsdam} & \quad 10.0 \pm 5.7 \text{ mcgal/y} \\
\text{Moscow} & \quad 11.1 \pm 2.4 \text{ mcgal/y} \\
\text{Novosibirsk} & \quad 9.0 \pm 3.0 \text{ mcgal/y}
\end{align*}
\]

average \( 10.0 \pm 2.3 \text{ mcgal/y} \)

Having in mind to use these results for statistical processing of a large amount of data, the measurements made in Potsdam and, Novosibirsk were reduced to Ledovo. Since the differences in gravity values between these points are known with great accuracy, this reduction was made practically without loss of accuracy. Fig. 4 shows results of these observations.

Assuming the observations to be equally accurate, their dependence on time was approximated by the parabola of the 2nd order. The reality of this approximation is confirmed by the relation of errors. If we consider the whole amount of measurements as the distribution of accidental errors, then the error of weight unit shall be \( \pm 11.2 \text{ mcgal} \). If this error is calculated by deviations from the approximating curve, then it is reduced to \( \pm 7.3\text{mcgal} \).

The obtained data, therefore, allows to presume, that at the points in Ledovo, Potsdam and Novosibirsk in the time interval 1976-1981 identical gravity changes were observed, which in the first approximation can be considered as quasiperiodic with the period of the order of 5 years and the amplitude of about 20 mcgal.

It is obvious that this amount of observations is insufficient either for the determination of the period, or for the determination of their amplitude. Therefore, to check the obtained result after the comparison of absolute gravimeters in Sèvres in October 1981, it would be useful to start systematic repeated measurements of the absolute value of gravity at the same points in the USA by the Faller and Hummond instruments, by Prof. Sakuma's instrument in Sèvres, by the Italian gravimeter at one of the points in the
south of Europe, by the Soviet instrument at Ledovo and Novosibirsk, in Japan at one or two points by the instruments constructed by the Japanese colleagues.

To facilitate future interpretation of results the measurements should be synchronised. Since the rate of gravity changes does not exceed 10-12 mgal/y and the accuracy of measurements by absolute gravimeters is within 4-7 mgal, their synchronisation within 2-3 months shall be sufficient.

August 1981
Moscow

References

1. Boulanger J.D., Gusev N.A. Gravity value at the international gravimetric point "Ledovo" (Moscow). In: "Results of high-precision gravimetric changes", Sov. Radio, M., 1977, pp. 7-11.
### Table 1

**Gravity changes in Europe**

<table>
<thead>
<tr>
<th>NN</th>
<th>Point</th>
<th>Gravity changes during the interval of 1958-1968</th>
<th>Gravity changes during the year</th>
<th>Gravity changes during the interval of 1968-1974</th>
<th>Gravity changes during the year</th>
<th>Gravity changes during the interval of 1958-1974</th>
<th>Gravity changes during the year</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Tallin</td>
<td>-</td>
<td>-</td>
<td>+ 12 ± 43</td>
<td>+ 2.0 ± 7.2</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>2.</td>
<td>Warsaw</td>
<td>- 120 ± 104</td>
<td>- 12.0 ± 10.4</td>
<td>+ 33</td>
<td>+ 5.0 5.7</td>
<td>- 87 ± 101</td>
<td>- 4.8 ± 5.6</td>
</tr>
<tr>
<td>3.</td>
<td>Krakow</td>
<td>-</td>
<td>-</td>
<td>- 29</td>
<td>- 4.8 5.8</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>4.</td>
<td>Prague</td>
<td>+ 80</td>
<td>+ 8.0 9.5</td>
<td>- 19</td>
<td>- 3.2 5.7</td>
<td>+ 61</td>
<td>92</td>
</tr>
<tr>
<td>5.</td>
<td>Budapest</td>
<td>+ 110</td>
<td>- 11.0 11.4</td>
<td>+ 7</td>
<td>+ 1.2 5.8</td>
<td>- 103</td>
<td>111</td>
</tr>
<tr>
<td>6.</td>
<td>Bucarest</td>
<td>- 10</td>
<td>- 1.0 13.6</td>
<td>+ 5</td>
<td>+ 0.8 7.5</td>
<td>- 5</td>
<td>132</td>
</tr>
<tr>
<td>7.</td>
<td>Sofia</td>
<td>+ 20</td>
<td>+ 2.0 12.1</td>
<td>- 29</td>
<td>- 4.8 9.2</td>
<td>-</td>
<td>9 112</td>
</tr>
</tbody>
</table>

Average: $|\Delta| = 6.7 \text{ mcgal}$  
$m = \pm 10.2 \text{ mcgal}$  

Average: $|\Delta| = 3.2 \text{ mcgal}$  
$m = \pm 7.0 \text{ mcgal}$  

Average: $|\Delta| = 2.9 \text{ mcgal}$  
$m = \pm 5.4 \text{ mcgal}$
<table>
<thead>
<tr>
<th>NN</th>
<th>Point</th>
<th>$g$ in IGSN-71 system</th>
<th>Type of measurements</th>
<th>New value of $g$</th>
<th>Difference $\Delta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Potsdam S-13</td>
<td>981 261 371 ± 17</td>
<td>Transmission from Ledovo by relative methods</td>
<td>981 261 420 ± 12</td>
<td>+ 49 ± 20</td>
</tr>
<tr>
<td>2.</td>
<td>Potsdam S-13</td>
<td>981 261 371 ± 17</td>
<td>Measurements by GABL instrument</td>
<td>981 261 416 ± 9</td>
<td>+ 45 ± 19</td>
</tr>
<tr>
<td>3.</td>
<td>Helsinki A</td>
<td>981 900 590 ± 19</td>
<td>Transmission from Potsdam by relative methods</td>
<td>981 900 632 ± 20</td>
<td>+ 35 ± 28</td>
</tr>
<tr>
<td>4.</td>
<td>Sevres A$_3$</td>
<td>980 925 880 ± 14</td>
<td>Measurements by GABL instrument</td>
<td>980 925 929 ± 6</td>
<td>+ 49 ± 15</td>
</tr>
<tr>
<td>5.</td>
<td>Sevres A$_3$</td>
<td>980 925 880 ± 14</td>
<td>Measurements by Sakuma instrument</td>
<td>980 925 926 ± 7</td>
<td>+ 46 ± 16</td>
</tr>
</tbody>
</table>

Average: $\bar{\Delta} = +45 \pm 2.6$ mcgal
## Table 3

Gravity changes in Australia

<table>
<thead>
<tr>
<th>NN</th>
<th>Point</th>
<th>Gravity at GABL points in IGSN-71 system</th>
<th>from GABL determinations</th>
<th>Δ</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>mcgal</td>
<td>mcgal</td>
<td>mcgal</td>
</tr>
<tr>
<td>1.</td>
<td>Sydney</td>
<td>979 637 586 ± 21</td>
<td>979 637 616 ± 9.1</td>
<td>+ 30 ± 23</td>
</tr>
<tr>
<td>2.</td>
<td>Alice</td>
<td>978 630 781 44</td>
<td>978 630 782 7.7</td>
<td>1 45</td>
</tr>
<tr>
<td>3.</td>
<td>Darwin</td>
<td>978 300 915 30</td>
<td>978 300 929 7.6</td>
<td>14 31</td>
</tr>
<tr>
<td>4.</td>
<td>Perth</td>
<td>979 403 631 24</td>
<td>979 403 688 8.0</td>
<td>53 25</td>
</tr>
</tbody>
</table>

Average: \[ \Delta = +24 \pm 11 \text{ mcgal} \]
Absolute Measurements with the IMGC transportable gravity meter within 1979 and 1981

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** Istituto Miniere e Geofisica Appl., Univ. Trieste, Cont. N° 68

Abstract.

Results of measurements performed with the transportable absolute gravity meter developed by the Istituto di Metrologia "G. Colonnetti" are presented and discussed in the framework of geodetical and geophysical projects.

Nineteen new absolute stations have been established, thirteen of them in Europe and six in the U.S.A. The average net uncertainty amounts to 11 μGal.

1. Introduction.

The accuracy (about 10 μGal) which may be achieved by the use of modern transportable absolute gravity meters offers a new tool for geophysical and geodetical studies.

The first need of geodetic metrology is the definition of a standardized gravity scale.

A suitable solution to this problem has been given with the International Gravity Standardization Net '71. By means of absolute gravimeters it is possible to improve the IGSN 71, to control its scale and linearity and to contribute to the definition of a new world gravity net mostly based on absolute measurements.

Many are the advantages of a net of this kind. We will mention just the relative short time in which an absolute site can be established (5 days or less), the freedom of the gravity datum from a "gravity system" (Potsdam, IGSN 71, ...), the possibility to have a gravity reference not related to relative measurements, which allows to study non linear terms in the calibration curves.
of relative gravimeters.

With this regard, 14 Absolute sites, measured with IMGC apparatus, form the Absolute European Calibration Line, whereas 5 sites measured in the U.S.A. contributed to the establishment of the North American calibration line.

A second need of geodetic metrology is concerned with national gravity nets. These are of primary importance for the establishment of the gravity datum and the gravity scale within a country.

Also in this field absolute gravimetry plays an important role. Contribution to the first order gravity nets of Austria, Italy, Switzerland and U.S.A. has been given with the aid of the IMGC absolute gravity meter.

Measurements of 11 sites, which will contribute to the Chinese fundamental gravity net are now in progress in the People's Republic of China.

Besides the geodetic aspect, absolute gravimetry can be used in establishing absolute control points for geophysical observations.

The freedom from calibration problems (which affect all relative gravimeters) makes an absolute gravity meter suitable mainly for studies on long term variations of the gravity field with the time.

With this regard the IMGC apparatus has been used in projects for studying gravity variations associated with earthquake activity (Alaska, Italy), Alpine uplift (Switzerland, Austria), Scandinavian shield uplift (Finland, Sweden).


2.1 In Europe.

Three gravity surveys have been performed in Switzerland between 1979 and 1980. Seven absolute stations have been established as a contribution to the Swiss fundamental net as well as to geodynamic investigations on the Alpine uplift. Two sites (Inter-
laken, Jungfraujoch) form the end points of a calibration line for field gravimeters. Two other sites (Erig and Chur) are located in the area of the most negative gravity anomalies and highest uplift encountered in Switzerland. Two stations are located under the S. Gothard tunnel. These four ones will serve as reference stations for more extended gravity nets for studying non periodic secular gravity variations associated with the Alpine uplift. Finally, the reference site in Zurich has been measured as well.

Four absolute gravity stations have been established in 1980 to contribute to the Austrian fundamental gravity net and to geodynamic investigations of the Alpine uplift. The stations are distributed equally over Austria, considering the close foreign absolute gravity stations in München (Germany), Chur (Switzerland) and Hoszeg (Hungary).

Altenburg (in the Bohemian massif, the most stable geologic unit in the vicinity of the Eastern Alps) and Kremsmunster (in the molasse of the northern Alpine foreland) are the end points of a calibration line for field gravimeters. The site at Penk is located within the shistcover of the Tauernwindow, the area of highest uplift rates in Austria. In Graz (southeastern rim of the Alps) the bedrock is formed by pleozoic formations. These stations will serve as reference sites for studying non periodic secular gravity variations associated with the Alpine uplift.

In 1979 and 1981 two absolute reference have been established in Italy for geophysical observations. The first one (Trieste) serves as reference for studies on gravity variations associated with the seismicity of the Friuli area. The second one (Palazzo al Piano) will be used as reference for studies on variations of the gravity field associated with the exploitation of the Travale geothermal field (Tuscany).

2.2 In the U.S.A.

Two campaigns have been carried out in the United States
of America in 1977 and 1980 respectively. The first one consists of six sites (Bedford, Denver, Bismarck, Alamogordo, San Francisco, Miami) as contribution to the improvement of the national base net. Six sites have been established during the second campaign, too. Four of them (Great Falls, Sheridan, Alamogordo, Mc Donald Obs.) contribute to the establishment of an high accuracy calibration line. The site in Anchorage will serve as reference for studies on gravity variations associated with earthquake activity in Alaska.

Tab 1 to 4 summarize the results of the measurements.

In Fig. 1, 2 schematic position maps of the absolute sites are depicted.

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**Fig 1 - Measurements in U.S.A.**  
**Fig 2 - Measurements in Europe**
Tab. 1 Absolute Measurements in Switzerland

<table>
<thead>
<tr>
<th>Site</th>
<th>Date</th>
<th>Nm</th>
<th>$\tilde{\Theta}$</th>
<th>unc.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Zürich</td>
<td>June 1978</td>
<td>100</td>
<td>980 647.873</td>
<td>.010</td>
</tr>
<tr>
<td>Chur</td>
<td>June 1978</td>
<td>100</td>
<td>453.815</td>
<td>.011</td>
</tr>
<tr>
<td>Interlaken</td>
<td>Aug 1979</td>
<td>132</td>
<td>305.463</td>
<td>.010</td>
</tr>
<tr>
<td>Jungfraujoch</td>
<td>Aug 1979</td>
<td>114</td>
<td>979 900.706</td>
<td>.010</td>
</tr>
<tr>
<td>Brig</td>
<td>Sep 1979</td>
<td>114</td>
<td>980 407.493</td>
<td>.010</td>
</tr>
<tr>
<td>St. Gottard 3</td>
<td>March 1980</td>
<td>100</td>
<td>310.864</td>
<td>.011</td>
</tr>
<tr>
<td>St. Gottard 4</td>
<td>March 1980</td>
<td>102</td>
<td>253.265</td>
<td>.010</td>
</tr>
</tbody>
</table>

Tab. 2 Absolute Measurements in Austria

<table>
<thead>
<tr>
<th>Site</th>
<th>Date</th>
<th>Nm</th>
<th>$\tilde{\Theta}$</th>
<th>unc.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Graz</td>
<td>Aug. 1980</td>
<td>110</td>
<td>980 715.532</td>
<td>.012</td>
</tr>
<tr>
<td>Altenburg</td>
<td>Aug. 1980</td>
<td>102</td>
<td>867.346</td>
<td>.012</td>
</tr>
<tr>
<td>Kremsmünster</td>
<td>Aug. 1980</td>
<td>107</td>
<td>741.228</td>
<td>.012</td>
</tr>
<tr>
<td>Penk</td>
<td>Aug. 1980</td>
<td>112</td>
<td>467.802</td>
<td>.012</td>
</tr>
</tbody>
</table>

Tab. 3 Absolute Measurements in Italy

<table>
<thead>
<tr>
<th>Site</th>
<th>Date</th>
<th>Nm</th>
<th>$\tilde{\Theta}$</th>
<th>unc.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Trieste</td>
<td>Oct. 1979</td>
<td>105</td>
<td>980 630.496</td>
<td>.010</td>
</tr>
<tr>
<td>Palazzo P.</td>
<td>July 1980</td>
<td>105</td>
<td>391.576</td>
<td>.010</td>
</tr>
</tbody>
</table>

Tab. 4 Absolute Measurements in U.S.A.

<table>
<thead>
<tr>
<th>Site</th>
<th>Date</th>
<th>Nm</th>
<th>$\tilde{\Theta}$</th>
<th>unc.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Boulder</td>
<td>May 1980</td>
<td>90</td>
<td>979 608.498</td>
<td>.011</td>
</tr>
<tr>
<td>Alamogordo</td>
<td>June 1980</td>
<td>101</td>
<td>139.584</td>
<td>.012</td>
</tr>
<tr>
<td>Mc Donald Obs.</td>
<td>June 1980</td>
<td>109</td>
<td>978 920.097</td>
<td>.011</td>
</tr>
<tr>
<td>Sheridan</td>
<td>June 1980</td>
<td>105</td>
<td>980 209.007</td>
<td>.011</td>
</tr>
<tr>
<td>Great Falls</td>
<td>June 1980</td>
<td>103</td>
<td>497.442</td>
<td>.010</td>
</tr>
<tr>
<td>Anchorage</td>
<td>June 1980</td>
<td>134</td>
<td>981 923.998</td>
<td>.010</td>
</tr>
</tbody>
</table>
3. Discussion of the results.

After the two sets of measurements in the U.S., some important results have been achieved.

First of all it is our opinion that comparison of transportable absolute gravity meters are significative only in field conditions. Environmental or internal effects may be active on a site rather than on another one.

Illustrative at this regard is the case of Alamogordo, New Mexico. A first set of measurements have been made in 1977. Results of repeated measurements carried out in Bedford, Massachusetts, at the beginning and at the end of the U.S. survey had confirmed a good behavior of the instrument, excluding that the internal calibration had been modified.

However measurements performed in 1980 with the AFGL apparatus indicated a significative discrepancy (80 μGal).

In order to solve this ambiguity, simultaneous measurements with both instruments were organized in 1980. The measurements in question were carried out in three days (May 31, June 2, 3). The results obtained with the IMGC instrument on May 31 are quite different from those of June 2 and 3. In fact the results of May 31 are in very good agreement with those of 1977, whereas the results of June 2 and 3 indicate a better agreement with AFGL instrument.

In an attempt to find an environmental condition that could have affected the 1977 measurements and those of May 31, 1980 it was found out that in both period a system for gyroscope testing was operating a few meter apart and shurthed off on June 2 and 3. As the system was of a classified character it was not possible to have the information necessary for a thorough understanding of the interaction between the system and the gravity meter.
Tab. 5 Synopsis of Alamogordo measurements.

<table>
<thead>
<tr>
<th>Instr.</th>
<th>date</th>
<th>g</th>
</tr>
</thead>
<tbody>
<tr>
<td>IMGC</td>
<td>1977</td>
<td>979</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.513</td>
</tr>
<tr>
<td>IMGC</td>
<td>May 31 1980</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>.516</td>
</tr>
<tr>
<td>IMGC</td>
<td>J. 2-3 1980</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>.584</td>
</tr>
<tr>
<td>AFGL</td>
<td>1979</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>.600</td>
</tr>
</tbody>
</table>

The tide correction to be applied to absolute measurements is a matter of discussion. Being a correction of relevant amplitude (even more than 200 μGal ptp) an accurate computation of it is very important.

Usually theoretical computed correction curves are applied. The accuracy of the most common computer programs is of the order of few μGal, which makes this correction suitable for the use with absolute measurements.

A more delicate problem is concerned with the amplification factors which in the most part of the sites is unknown. The use of a recording gravity meter could provide a suitable experimental curve which might be used to compute the correction, or at least, to control the rightness of the theoretical curve.

However the variations of the $\delta$ factor with respect to the average (1.17) are of the order of some percent units. Considering that the average standard deviation for a single absolute measurement is of the order of 30 μGal, the effect of errors on the $\delta$ factor are not too important.

Bigger deviation of the sperimental earth tide curve from the theoretical one may be detected by means of absolute measurements, too.

In Fig 3 to 6 theoretic earth tide curves, computed for some sites, are depicted together with non corrected absolute measures as well.
The measured values are the mean values of sets of 15-20 absolute measurements, with average standard error of 8 µGal.

The reported examples show, generally, a good agreement between computed and measured earth tide curves. If there are differences of the $S$ factors they are not significative with regard to error terms of the absolute measurements. Significant variations, if any, should be detected with the aid of these graphs.

During the course of the measurements enviromental conditions (microseismic noise, electric noise, temperature variations, ...) or small troubles shooting the gravity meter (wearing of some mechanisms, electrostatic charges on the cube corner, ...) can alter slightly the results introducing small systematic errors.

An example of this is reported in Fig. 6 where one set of measurements (12 data) appears affected by a small systematic error, whereas the rest of the data (105) fit very well the theoretical curve.

Attention should be paid, analyzing the data, to situations of this kind.
Fig. 3 - Earth tide curve: NN Donald Obs, June 5-7, 30
* computed  ○ measured

Fig. 4 - Earth tide curve: Alamagordo June 2-3, 30
* computed  ○ Measured
Figure 5 - Earth tide curve: Cnur June 15-16, 1978
- computed
○ measured

Figure 6 - Earth tide curve: Trieste Oct. 5-6, 1979
- computed
○ measured
References


ABSTRACT AND RELATIVE GRAVITY MEASUREMENTS IN SWITZERLAND
WITH SPECIAL EMPHASIS ON A NEW SWISS NATIONAL GRAVITY NET

by Fischer, W., Kahle, H.-G. and I. Marson

Abstract

Absolute gravity measurements have been carried out between 1978 and 1980
at 7 stations in Switzerland with the transportable absolute gravity apparatus

Relative gravity measurements have been performed in Switzerland since
1962 in order to establish a new Swiss National Gravity Net. During the period
1975-1979, the gravimetric activities have been devoted to the completion of a
new Bouguer gravity map of Switzerland. A special network connecting the 7 ab-
solute sites and including the calibration line Interlaken - Jungfraujoch has
been measured 1980-1981 in order to determine the scale of two LaCoste & Rom-
berg G-gravimeters and one D-model. In addition, this network has allowed some
valuable investigations on the periodic errors of the two G-meters in question.

Measurements of non-periodic secular gravity variation are planned in the
Rhine valley near Chur and Sargans, where earlier investigations have shown a
pronounced minimum of the isostatic anomalies as well as the highest uplift
rates of the country. This project has been started in autumn 1981. It is in-
tended to connect this gravity line with a similar profile (Munich - Verona) in
the Brenner area, Austria, which has been established by the Bayerische Kommiss-
sion für die Internationale Erdmessung, Germany.

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1. Absolute and relative gravity measurements in Switzerland

Intensive gravimetric work has been carried out in Switzerland in the last years. It has been part of geodynamic investigations on the Alpine uplift and has served at the same time as a base for the establishment of a new Swiss National Gravity Net (SNGN).

1.1 Absolute gravity measurements

A project of measuring absolute gravity in Switzerland has been initiated in 1978 in a joint effort by the ETH Zürich, the University of Trieste and the Istituto di Metrologia "G. Colonnetti", Torino. Seven absolute gravity stations of high accuracy have been established until now. They have been described by Marson et al. (1981) and Kahle et al. (1981).

At each station the transportable absolute gravity meter developed by the Istituto di Metrologia "G. Colonnetti" has been used. In this type of instrument the gravity measurement is based on the observation of the symmetrical free rise and fall of a body in the earth's gravitational field. About 100 independent measurements have been taken at each station. Earth tidal corrections have been applied to each single value using an amplification δ factor of 1.14. For the reduction of the mean value of gravity to the reference level (floor) the measured vertical gravity gradient has been applied. The reduced gravity values of each station have a net uncertainty of ± 10 μgal.
Fig. 1: Absolute gravity stations in Switzerland on a generalized tectonic map of Switzerland provided by the Geological Institute ETH Zürich

Fig. 1 displays the location of the Swiss absolute stations on a generalized tectonic map provided by the Geologisches Institut of ETH Zürich. The first station, Zürich, measured 1978, is intended to serve as a fundamental point of the Swiss National Gravity Net. The station Chur, observed in the same year, is situated on the Rhine-Rhône line where the lowest values (-180 mGal) of the Bouguer gravity anomaly and the highest uplift rates (+1.7 mm/y) have been noted. In 1979, the stations Interlaken and Jungfraujoch were established as endpoints of a nearly "vertical" gravity calibration line covering a gravity range of about 605 mGal (Klingelé and Kahle, 1981). The station Brig, observed also in 1979, is located on the Rhine-Rhône line. Finally there was a unique opportunity in 1980 to measure absolute gravity within the Gotthard highway tunnel for geodynamic purposes prior to the official opening of the tunnel. The two stations Bázberg and Guspisbach are situated in the Aar massif and in the Gotthard massif, respectively.
1.2 Relative gravity measurements

The efforts for the establishment of a new Swiss National Gravity Net (SNGN) were started already in 1959, when the Swiss Geodetic Commission planned a 1st order gravity net. The measurements for this network were performed during the period 1962-1966 using the gravity meter Worden No 472. The network represented in fig. 2 is connected to the earlier fundamental gravity net. Unexpected gaps in the connecting points were considered to be due to strains within the existing network and to thermal effects disturbing the stability of the Worden gravity meter.

---

Fig. 2: Swiss National Gravity Net (SNGN)

Therefore a control network was measured in 1968, using the three LaCoste and Romberg gravity meters G 24, G 54 and G 115 (see fig. 2).
In connection with the compilation of the new Swiss gravity map (Klingelé and Olivier, 1980), another control network was measured in 1976, consisting of 14 stations which were connected by triangles. The two LaCoste and Romberg gravity meters G 317 and G 369 were used simultaneously.

A comparison of the different networks revealed that the scales of the various gravity meters represent an element of serious uncertainty. Therefore the 7 absolute gravity determinations within the network are of great importance for the final adjustment (see 1.1). In order to connect the absolute gravity stations to the existing gravity network relative gravity measurements have been performed between the absolute sites and the adjacent gravity stations. At this occasion also the connections between neighbouring absolute stations have been measured. This has been done in 1980 and 1981, using simultaneously the two LaCoste and Romberg gravity meters G 317 and G 514 (see fig. 2).

Apart of these relative gravity measurements special attention has to be paid to the measurements carried out for the new Bouguer gravity anomaly map of Switzerland (Klingelé and Olivier, 1980). More than 2000 gravity stations have been determined all over the country which corresponds to a station density of about 1 gravity station per 20 km².

1.3 Secular gravity variations

Secular gravity changes can be determined either by absolute or by relative gravity measurements. From a comparison of recent relative measurements (see 1.2) with earlier measurements (e.g. the fundamental gravity net), however, no significant changes can be deduced because of the uncertainty of the earlier gravity observations.

As already mentioned in chapter 1.1, several absolute gravity determinations of high accuracy have been completed in the last years. Especially the gravity values of Chur, Brig, Bäzberg and Guspisbach are intended to serve as a base for deducing secular gravity variations by comparing them with repeated determinations in due time.
In addition a detailed investigation is planned in the Rhine valley near Chur and Sargans by using relative gravity measurements. In this area the uplift rates themselves as well as their lateral gradients are relatively large. At the same time the Bouguer gravity anomalies as well as the isostatic anomalies show a pronounced minimum and a relatively large lateral gradient near Chur. On a N-S profile 6 gravity stations with small gravity differences have been selected to serve as a uniform-gravity line for detecting secular changes in gravity. The first measurements have been commenced in autumn 1981. A similar profile Munich–Verona has been established by the Bayerische Kommission für die Internationale Erdmessung, Germany, and will be connected in the Brenner area, Austria, to the Swiss line by means of relative gravity measurements.

2. The new Swiss national gravity net (SNGN)

The existence of 7 absolute gravity stations in Switzerland will allow that a well-determined new gravity net can be established over the entire country. The 7 absolute gravity values define both the gravity level as well as the scale of the network. The gravity values of all stations of the network will be determined by using a least squares adjustment. In a first step the scale factors of the recently used gravity meters are deduced from the gravity calibration line Interlaken–Jungfraujoch as well as from a network connecting 6 absolute gravity stations (fig. 2).

2.1 The gravity calibration line Interlaken–Jungfraujoch

A computer program developed for the adjustment of SNGN has been checked by using relative gravity measurements on the gravity calibration line Interlaken–Jungfraujoch (Fischer, 1981). Based on the absolute gravity values at the two endpoints Interlaken and Jungfraujoch, the unknown scale corrections became by a common adjustment:

for D 16: M 16 = + 0.32 ± 0.07 %,
for G 317: M 317 = + 0.13 ± 0.07 %,
for G 514: M 514 = - 0.11 ± 0.07 %.
2.2 The connection net of 6 absolute gravity stations

As mentioned in chapter 1.2 the absolute gravity stations have been connected to the neighbouring stations of the gravity network by relative gravity measurements (fig. 2). At the same time the gravity differences between 6 absolute gravity stations have been measured using the two LaCoste and Romberg gravity meters G 317 and G 514.

Fig. 3: Connection net including the gravity calibration line Interlaken-Jungfraujoch

The corresponding connection net is shown in fig. 3. In addition the gravity calibration line Interlaken-Jungfraujoch with the 5 intermediate points is indicated. The two stations Bäzberg and Guspisbach within the Gotthard highway tunnel were tied prior to the beginning of the traffic to the point Göschenen of the fundamental gravity net. The spatial distances between the sites are always less than 100 km but on several lines Alpine passes have to be crossed. The connection to Zürich can be made by using highways for transportation.

A vertical section of the connection net is sketched in fig. 4 in order to give an idea of the gravity ranges. The ordinates are the gravity values, increasing downwards so that the highest station Jungfraujoch of the gravity calibration line Interlaken-Jungfraujoch is also at the top. With this connection net, including the gravity calibration line, a gravity range of nearly 800 mgal is covered. Thus the entire gravity range within Switzerland can be covered, except of the area of Basel where gravity values of about 980 760 mgal are observed.
Fig. 4: Vertical section of the connection net, gravity values increasing downwards.

2.3 The adjustment of the connection net

A total of 246 gravity observations have been used for an adjustment in which the gravity values of all stations and the scale corrections of the two G meters G 317 and G 514 have been treated as unknowns. The 6 absolute gravity measurements have also been treated as observations. Each observation has been introduced with the weight 1, corresponding to an m.s.e. of 10 μgal.

The result of the adjustment has been somehow disappointing since it has shown an r.m.s. error of 18 μgal, which is nearly twice the m.s.e. a priori. This great amount is partly due to obvious discrepancies in some absolute sites between the absolute gravity value and the corresponding value determined by the relative measurements within the connection net. The discrepancies correspond to some extent to the residuals v of the 6 absolute gravity measurements which are represented in fig. 5. A statistical test of the ratios v/m(v) has shown some significant disagreements and has forced us to reject the used mathematical model. As it is clearly seen in fig. 5 the most striking station is Chur. Special attention has also to be payed to the two Gotthard stations Bälzberg and Guspiisbach, which are only about 7 km apart from each other. At first sight no explanation has been found for these discrepancies.
The scale corrections deduced by the adjustment are:

for G 317: \( M_{317} = +0.00 \pm 0.06 \% \),
for G 514: \( M_{514} = -0.20 \pm 0.07 \% \).

They are both about 0.1 \% smaller than those found on the gravity calibration line Interlaken-Jungfraujoch (see 2.1). Although the differences are within the m.s.e. due to the two independent determinations the latter values seem to be rather erratic, taking into consideration the various residuals of the 6 absolute values (see fig. 5).

2.4 Periodic errors

Periodic errors of the gravity meters as suggested by several authors (Kiviniemi 1974), (Torge and Kannrieser 1980) have initially not been taken into account because of the great amounts of discrepancy.

In a second step an attempt has been made to compare the residuals with an error curve based on the period 71 mgal which corresponds approximately to one full turn of the measuring screw of the G meters. Fairly good agreement has been found with a curve of 50 ugal amplitudes. Therefore periodic errors of such high amount can be proposed as a first provisional result.

![Diagram](image)

Fig. 5: Residuals \( v \) of 6 absolute gravity measurements from a common adjustment including relative gravity measurements with G 317 and G 514.
Since a relatively large gravity range of about 390 mgal is encountered in the connection net (Zürich-Guspisbach) - and of even about 605 mgal along the gravity calibration line Interlaken-Jungfraujoch - (see fig. 4) it is not sufficient to use an approximate period of 71 mgal, but it is necessary to utilize the effective period of the cyclic error. Considering for each gravity meter the screw factors valid in our gravity range the exact periods are:

for G 317: period = 70.94 \cdot 1.0569 = 74.98 mgal,
for G 514: period = 70.94 \cdot 1.0187 = 72.27 mgal.

It is seen that the small difference in the periods causes a shift of 1/4 period in the error curves after a gravity change of 500 mgal (approximately 7 periods). It is, therefore, doubtful to use a common adjustment including all gravity measurements. Hence, separate adjustments including measurements of only one G meter have been carried out. The corresponding residuals of the 6 absolute gravity measurements are shown in fig. 6 and 7. They fit better with a respective error curve than the residuals of the common adjustment in fig. 5. Since the residuals in Interlaken are nearly zero the phase angle of the error curve is put zero at this station. The amplitude of the curve has been found to be about 30 µgal (see fig. 6 and 7).

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Fig. 6: Residuals $v$ of 6 absolute gravity measurements and best fitting curve of periodic error for the relative gravity meter G 317.

Fig. 7: Residuals $v$ of 6 absolute gravity measurements and best fitting curve of periodic error for the relative gravity meter G 514.
3. Conclusions

In order to establish a new Swiss National Gravity Net (SHGN) a relatively dense network of relative gravity measurements has been measured in the last years. In addition absolute gravity measurements have been performed for several purposes at 7 different sites.

Firstly, the check of the scale values of three LaCoste and Romberg gravity meters (D 16, G 317 and G 514) resulted in small but distinct scale corrections.

In a common adjustment of all relative and 6 absolute gravity measurements in a connecting net, significant discrepancies between absolute and relative measurements are observed. After a comparison with theoretical periodic errors it is tempting to suppose that the two G meters are affected by at least one periodic error of the period 70.94 counter units and an amplitude of about 30 ugal. No special investigations have yet been made for the D meter D 16.

It is planned to measure the connecting net of the 6 absolute gravity stations also by means of D meters. As it is shown in fig. 4, all the gravity differences between the 7 stations of this net are smaller than 200 mgal and are thus capable to be measured directly with this type of gravity meters, except the difference Zürich-Göschenen which has to be subdivided into two sections. It is intended to perform the measurements in summer 1982 using the D 16 and a second D meter which is being purchased at this time. Other D meters are highly welcome to participate in this project.

4. Acknowledgments

Financial support for the absolute gravity measurements provided by the Swiss National Science Foundation, contracts No 2.871-0.77 and No 2.629.0.80 and by the Swiss Geodetic Commission is gratefully acknowledged. The manuscript was typed by Mrs. A. Cagienard.
References


CALIBRATION OF LA COSTE-ROMBERG GRAVITY METERS
MODEL G AND D

by E. Kimmelser and W. Torge *


Abstract:

Between 1967 and 1980 the Institut für Theoretische Geodäsie, Universität Hannover, carried out long range gravity measurements between 25 stations in North and Central Europe (gravity range 9.307 ... 9.826 ms⁻²) in order to determine linear and quadratic as well as long-period calibration terms for LaCoste-Romberg gravity meters. Significant changes in the calibration functions have been found only due to service or repair work carried out at the manufacturer. Between 1976 and 1981 special calibration lines with gravity ranges of 0.1, 2, 20, 200 and 2000×10⁻² ms⁻² have been used for the detection of periodic calibration terms. If the periodic terms are taken into account, a significant accuracy increase is obtained.

1. Introduction:

A detailed knowledge of the calibration curve of the gravity meters is important for high precision measurements. The instrumental calibration function, as supplied for the LaCoste-Romberg (LCR) gravity meters by the manufacturer as calibration table, has an accuracy of 10⁻³ to 10⁻⁴ (KANNIESER et al. 1978, WENZEL and TORGE 1978). Residual linear and quadratic terms caused by non-linearity of the lever system, especially affect large gravity differences. Graduation errors and eccentricities in the screw and the gears influence mainly small gravity differences.

The present paper reports about investigations carried out at the Institut für Theoretische Geodäsie, Universität Hannover, to determine the LCR-gravimeter calibration functions with μGal-order accuracy (1 μGal = 10⁻⁵ ms⁻²).

2. Mathematical Model

For the development of a correction model of the manufacturer's calibration function, polynomials and Fourier series have been applied. The polynomial describes the non-linearity of the double reduction lever system in the LCR-gravimeters, and the Fourier series models the graduation errors and the eccentricities in the screw and the gears.

\[ F(z_i) = F_o(z_i) + \sum_{k=1}^{\infty} \frac{k}{z_i^k} E_k + \sum_{n=1}^{\infty} A_n \cdot \cos(n \cdot z_i^{1-n}) \]

where

- \( z_i \) = raw gravity meter reading in i
- \( F_o \) = manufacturer's calibration function

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\[ \Delta \xi, j + v_{i, j} = g_i - g_j - E_1 (Z_i - Z_j) - E_2 (Z_i^2 - Z_j^2) + \]
\[ + \sum_{n=1}^{p} X_n \left( \frac{Z_i^{2 \pi}}{T_n} \cos \frac{Z_j^{2 \pi}}{T_n} - \frac{Z_j^{2 \pi}}{T_n} \cos \frac{Z_i^{2 \pi}}{T_n} \right) + \sum_{n=1}^{p} Y_n \left( \frac{Z_i^{2 \pi}}{T_n} \sin \frac{Z_j^{2 \pi}}{T_n} - \frac{Z_j^{2 \pi}}{T_n} \sin \frac{Z_i^{2 \pi}}{T_n} \right) + \]
\[ + D (t_i - t_j) \]
with the auxiliary parameters
\[ X_n = A_n \cdot \cos \phi_n, \quad Y_n = A_n \cdot \sin \phi_n, \]
\[ A_n = \sqrt{X_n^2 + Y_n^2}, \]
\[ \tan \phi_n = \frac{Y_n}{X_n} \]
and
\[ \Delta \xi, j = \text{gravity difference between } i \text{ and } j, \text{ corrected for earth tides and other known systematic effects, and converted with the manufacturer's calibration table} \]
\[ v_{i, j} = \text{residual of } \Delta \xi, j \]
\[ g_i, g_j = \text{adjusted gravity values of station } i, j \]
\[ E_1, E_2 = \text{linear, resp. quadratic calibration terms} \]
\[ D = \text{linear drift coefficient} \]
\[ t_i, t_j = \text{time of measurement in } i, j. \]
For the absolute stations the observation equation read as
\[ Z + v_i = g_i \]
with
\[ i = \text{observed absolute gravity value in } i \]
\[ \tilde{g}_i = \text{adjusted absolute gravity value in } i. \]

Differences of subsequent readings resp. observed absolute gravity values are introduced as observations in the least squares adjustment taking into account the empirical correlation of sequential gravity differences. The unknowns of the system are the gravity values, the linear and quadratic calibration terms, the auxiliary parameters of the periodic terms and linear parameters of the drift functions.

3. Results of the Calibration System North and Central Europe

This calibration system serves for determining of linear and quadratic as well as long-period calibration terms\(^+\). The sites of the 18 stations are shown in figure 1, short description are given in table 1. The 11 absolute stations have been established by CANNIZZO et al. 1978. The system includes the following stations of the International Gravity Standardization Net 1971 (IGSN71): 17981A, 18082A, 21629A, 21639A, 21590 O, 25174J, 28603A (Bureau Gravimétrique International, Toulouse). Stations number 31 and 35 have been established at a former calibration survey (TORGÈ et al. 1976) and 608 and 612 densify the northernmost part of the system (TORGÈ and KANNGIESER 1980).

Gravity observations in the calibration system include
- the absolute determination in Paris (SAKUMA 1976),
- absolute measurements along the European Absolute Calibration Line (CANNIZZO et al. 1976),

The data preprocessing comprised the transformation of the gravimeter readings to gravity units, using the manufacturer’s conversion tables or for the model D-instruments the constant calibration factor, and the tidal reduction. For the calculations of earth tides the CARTWRIGHT/TAYLER/EDDEN tidal potential development with 505 waves (CARTWRIGHT and TAYLER 1971, CARTWRIGHT and EDDEN 1973) has been used, including the time independent term \( M_0 S_0 \). The program has been developed by (WENZEL 1976). Regional tidal parameters for the main waves have been derived from the results of gravimetric earth tide observations (DUCARME and KÄRKIÄINEN 1980, GERSTENBACHER and GROTTEN 1976, MELCHIOR et al. 1976, TORGÈ and WENZEL 1977). The amplitude factors and the phase lags are given in table 2.

Common adjustments have been performed introducing 357 observed gravity differences and 12 absolute gravity values. The r.m.s. error of the absolute values of CANNIZZO et al. (1978) was estimated to \( \pm 10 \, \mu \text{Gal} \) and of SAKUMA (1976) to \( \pm 2 \, \mu \text{Gal} \). The relative measurements have been corrected because of the known periodic errors (chapter 4).

Except for LCR G 85, the first adjustment gave only significant linear calibration terms for each epoch (tab. 3). Significant changes in the calibration function have been found only after service or repair work carried out at the manufacturer, or when different gravity ranges have been used for the calibration. In G 85 a capacitive readout has been established in winter 1978/79, which procedure probably changed strongly the calibration. Adjustment with a linear calibration term gives a rather large value for the calibration factor and a r.m.s. error of the observed gravity differences of \( \pm 42 \, \mu \text{Gal} \). This value reduces to \( \pm 18 \, \mu \text{Gal} \) when introducing a quadratic term. In the final adjustment overall calibration terms have been determined, when there was no service or repair work at the

\(^+\) The determination of the long-period terms is described in section 4.
gravity meter. The long-wave dependence between the calibration and the gravity ranges could be modelled by quadratic terms. The linear calibration terms are determined with a r.m.s. error of $\pm 1$ to $\pm 2 \times 10^{-5}$ by means of the absolute measurements of CANNIZZO et al. (1978) and SAKUMA (1976), tab. 3. The overall calibration factor of 0.14 does not deviate strongly from 1, but the large r.m.s. errors of the linear terms for the single epochs indicate that the calibration changes with the reset. This is confirmed by admitting different factors for each gravity interval observed in 1979, giving values between 0.99926 and 1.00025, with an average r.m.s. error of $\pm 1 \times 10^{-5}$ (TORGÉ and KANNGIESER 1980). If this effect is not taken into account, relative errors up to nearly $1 \times 10^{-3}$ may occur. Similar results were found by LAMBERT et al. (1979). Therefore the observations of LCR gravimeter D 14 were not used in the final adjustment. The calibration terms for 1967, 1972 and 1976 calculated using the absolute value of CANNIZZO et al. (1978) and SAKUMA (1976) deviate not significantly from those calculated by means of the IGSN71-gravity values (TORGÉ 1971, TORGÉ et al. 1976, WENZEL and TORGÉ 1978). The maximum difference reaches $1 \times 10^{-4}$. The linear factors of 1975 differ significantly, because the IGSN71-gravity value of Bodé deviates about 0.1 mGal from the absolute value.

The gravity values, their r.m.s. errors and the variance-covariance matrix of the station gravity values are given in table 1 and table 4. The accuracy of the absolute gravity stations has been increased from $\pm 10$ uGal to $\pm 5 \ldots $ uGal, as by the connecting relative measurements, now all absolute values contribute to the system level.

4. The Calibration System Hannover

The investigation of periodical terms of the calibration function has been based on the periods to be expected from the instrument’s construction. The crosshair for the reading of the gravimeter is moved by the gear box. The eccentricities and the graduation errors in the gears and the screw cause the periodic errors. The teeth ratios of the driving and driven gears lead to the periods in table 5 (HOKASALO 1971, KIVIINIEMI 1974, LACOSTE 1980). The periodic errors in the model G-gravity meters may reach amplitudes of 35 uGal and are the most important instrumental error source (HARRISON and LACOSTE 1978). In order to detect these periods by measurements along calibration lines, the lines should have a sufficiently large gravity difference, containing possible more than one period of the term reached for, and a sufficiently narrow subdivision.

Following these consideration and taking into account the periods theoretically expected, three calibration lines have been used at our investigations (tab. 5).

The vertical calibration line Hannover has been established in the staircase of a 19 storied university building (Callinstraße 13), see figure 6, in order to determine short-periodic terms. In the basement, 11 stations with gravity differences of 0.02 mGal and 11 stations with gravity differences of 0.2 mGal serve for the investigation of the 1 mGal-period for the model G and of the 0.1, 0.7 and 1.6 mGal-period for the model D gravimeters, while 23 stations with 1 mGal differences are used for the detection of the 3.9 and 7.9 mGal period for the model G and of the 3.2-mGal period for the model D gravimeters.

The Hannover-Harz calibration line (fig.2) utilizes the gravity variation with latitude and height (station Hannover no. 1: $l = 52^\circ 22'$, $h = 50$ m; station Torfhaus no. 560: $l = 51^\circ 48'$, $h = 785$ m). The 20 monumental stations of this 193 mGal-line have an average gravity difference of 10 mGal, thus allowing the investigation of the 35 and 70-mGal periods. With a LCR model D gravity meter, the line can be measured in a fixed position of the screw.
For the determination of long-periodic terms, the northern part of the European absolute calibration line (CANNIZZO et al. 1978) can be used, between the stations München and Hammerfest (fig. 1, closed circles). This calibration line comprises 10 absolute stations. For the investigation of model D gravity meters, two intermediate stations have been established between Vazza and Hammerfest: Oulu (according to personal information from J. MARSON and R.K. MCCONNELL 1979) and Inari. With the exception of München-Wiesbaden, all gravity differences between adjacent stations can now be observed with a model D instrument, without a reset.

Between 1976 and 1981, numerous observation have been carried out on the calibration lines, employing altogether 6 LCR model G and 2 LCR model D gravity meters. Table 5 gives some statistics about these surveys, including the r.m.s. errors of one observed gravity difference, coming out from different adjustments. The data preprocessing has been carried out as described in chapter 3.

A first accuracy estimation became available by separate adjustments for each gravity meter (tab. 6). Subsequently common adjustments introducing the observations of all instruments on one calibration line have been performed. A priori weights have been estimated according to the results of the separate adjustments. Linear and quadratic calibration terms were introduced with the values obtained in chapter 3. The r.m.s. errors of one observed gravity difference are given in table 6. For the determination of the periodic terms, a common adjustment introducing the observations of all instruments of all calibration lines have been performed. Linear and quadratic calibration terms were introduced as before and all theoretically expected periodic terms and the gravity values of the calibration stations were adjusted.

The quality of the calibration lines is documented by the accuracy of the adjusted gravity differences. Their r.m.s. errors vary between \( \pm 0.7 \) and \( \pm 1.5 \) (average \( \pm 1.1 \)) \( \mu \text{Gal} \) for the vertical calibration line Hannover, between \( \pm 1.0 \) and \( \pm 1.7 \) (average \( \pm 1.4 \)) \( \mu \text{Gal} \) for the Hannover-Harz calibration line and between \( \pm 3.3 \) and \( \pm 8.4 \) (average \( \pm 6.8 \)) \( \mu \text{Gal} \) for the northern part of the European absolute calibration line.

The significant periodic terms, found at the calibration lines, are given in table 7.

The amplitudes found at the LCR-gravity meters model G are

\[
\begin{align*}
&< 5 \ \mu\text{Gal} \text{ for the } 1 \ \text{mGal-period} \\
&< 2 \ \mu\text{Gal} \text{ for the } 3.9 \ \text{mGal-period} \\
&< 9 \ \mu\text{Gal} \text{ for the } 7.9 \ \text{mGal-period} \\
&< 8 \ \mu\text{Gal} \text{ for the } 15 \ \text{mGal-period} \\
&< 20 \ \mu\text{Gal} \text{ for the } 71 \ \text{mGal-period} 
\end{align*}
\]

For the 603 and the 1206 mGal-period, amplitudes up to 20 \( \mu \text{Gal} \) have been found, but due to the limited range of the absolute calibration line and the few measurements, these values might be uncertain.

The amplitudes for the model D instruments are

\[
\begin{align*}
&< 1 \ \mu\text{Gal} \text{ for the } 0.7 \ \text{mGal-period} \\
&< 2 \ \mu\text{Gal} \text{ for the } 1.6 \ \text{mGal-period} \\
&< 3 \ \mu\text{Gal} \text{ for the } 3.2 \ \text{mGal-period} 
\end{align*}
\]

The amplitudes are reduced significantly from model G to D meters, due to the change of the reduction ratio of the lever system (HARRISON and LACOSTE 1978). The 0.1 mGal-period could not be analyzed.
The calibration line Hannover-Harz (gravity range nearly 200 mGal) has also been established in order to determine linear and quadratic calibration terms for LCR-gravity meters model D in a fixed position of the reset screw. The analyzed calibration terms are given in table 8. Significant changes in the calibration have been found for both model D-instruments on this calibration line again, when the reset screw is brought to another position (chapter 3).

<table>
<thead>
<tr>
<th>instrument</th>
<th>linear calibration term and r.m.s.e.</th>
<th>quadratic calibration term and r.m.s.e.</th>
</tr>
</thead>
<tbody>
<tr>
<td>D 14</td>
<td>1.00026 ( \pm 5 )</td>
<td>-0.000021 ( \pm 2 )</td>
</tr>
<tr>
<td>D 23</td>
<td>1.00047 ( \pm 6 )</td>
<td>-0.000022 ( \pm 3 )</td>
</tr>
</tbody>
</table>

Tab. 8: Linear and quadratic calibration terms for LCR-gravity meters model D

From table 6 we recognize that the r.m.s. errors of the observed gravity differences have increased, from the separate to the common adjustments. The reason for the increase is the existence of periodic errors. Consequently, one common adjustment introducing the known linear, quadratic and periodic calibration terms has been carried out. The introduction of the non-linear calibration terms reduces the r.m.s. errors of the observed gravity differences, in many cases even exactly to the values obtained from the separate adjustments (tab. 6). The r.m.s. errors of the common adjustments reduced, on the average, from \( \pm 7 \) to \( \pm 6 \) \( \mu \text{Gal} \) on the vertical calibration line, from \( \pm 18 \) to \( \pm 13 \) \( \mu \text{Gal} \) on the Hannover-Harz calibration line, from \( \pm 23 \) to \( \pm 21 \) \( \mu \text{Gal} \) on the European absolute calibration line, by the inclusion of the periodic terms. The average influence of the periodic terms is \( \pm 4 \) \( \mu \text{Gal} \), \( \pm 12 \) \( \mu \text{Gal} \) and \( \pm 9 \) \( \mu \text{Gal} \), resp.

5. Conclusions

Linear calibration terms can be derived from presently available absolute gravity values with an accuracy of \( \pm 1 \) to \( \pm 2 \times 10^{-5} \). The long-wave dependence between the calibration and the gravity range has been modelled by a quadratic term, while short-period terms are described by Fourier series. Calibration measurements are necessary after each service or repair work. For two model D gravity meters, significant variations of the linear calibration factor were found at different reset positions, and quadratic terms have been analyzed in a fixed reset position.

By use of several LCR-gravity meters, gravimeter calibration lines up to 200 mGal can be established with \( \mu \text{Gal} \)-accuracy, and evaluated for the detection of periodic calibration
terms. For model G instruments, an average amplitude of 9 µGal has been found, while the average amplitude for model D gravimeters is only 2 µGal. By correction of the periodic errors, systematic discrepancies disappear nearly completely. Investigations about the transfer of the detected periodic terms to other high precision gravity networks, are under progress now. The preliminary results indicate a significant increase of the accuracy, even if the gravity range of the network is different from the range of the calibration lines.

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The German Research Society (Deutsche Forschungsgemeinschaft) generously sponsored the investigations, especially on the European absolute calibration line.

References:


![Fig.1: Calibration system North and Central Europe](image-url)
Fig. 2: Vertical calibration line Hannover

Fig. 3: Calibration line Hannover-Harz
<table>
<thead>
<tr>
<th>Station number</th>
<th>Station description</th>
<th>Gravity value (mGal)</th>
<th>r.m.s.e. (mGal)</th>
</tr>
</thead>
<tbody>
<tr>
<td>40</td>
<td>München A (Deutsches Geodätisches Forschungsinstitut, 1798 A) *</td>
<td>980 723.128</td>
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<td>111</td>
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<td>Wiesbaden (Landesvermessungszentrale) *</td>
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<td>151</td>
<td>Braunschweig (Physikalisch-Technische Bundesanstalt) *</td>
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<td>Hannover A (Universität, 2162 A)</td>
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<td>Göteborg (Chalmers Tekniska Högskola) *</td>
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<td>606</td>
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<td>982 090.753</td>
<td>± 6</td>
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<td>35</td>
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<td>± 10</td>
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<td>408</td>
<td>Oulu (Hotel Vaskuna)</td>
<td>982 255.205</td>
<td>± 6</td>
</tr>
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<td>31</td>
<td>Polic circle (south of Bodø)</td>
<td>982 552.221</td>
<td>± 13</td>
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<td>12</td>
<td>Bodø J (Airport, 15174 J)</td>
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<td>612</td>
<td>Inari (church)</td>
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<td>Hammerfest A (Brannstasjon, 2163 A) *</td>
<td>982 617.541</td>
<td>± 8</td>
</tr>
</tbody>
</table>

**Tab. 1:** Adjusted gravity values of the calibration system North and Central Europe

*) absolute station (CANNIII et al. 1978)

<table>
<thead>
<tr>
<th>Calibr. Station</th>
<th>Earth tide station</th>
<th>Diurnal waves</th>
<th>Semi diurnal waves</th>
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<tr>
<td>Kopenhagen</td>
<td>Aarhus</td>
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<td></td>
</tr>
<tr>
<td></td>
<td>Bodø</td>
<td>1984 10.12 1.1724 -0.18</td>
<td>1.1597 ± 0.63</td>
</tr>
<tr>
<td></td>
<td>Blue-</td>
<td>1.2238 9.19 0.812 ± 0.1543 0.3899 0.50 0.1071 0.97 0.864 2.18</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Schwei</td>
<td>1.182 2.45 1.2104 1.83 1.230 1.04</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Bodø</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Blue-</td>
<td>1.2238 9.19 0.812 ± 0.1543 0.3899 0.50 0.1071 0.97 0.864 2.18</td>
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<td>Schwei</td>
<td>1.182 2.45 1.2104 1.83 1.230 1.04</td>
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<td></td>
<td>Frankf. (Hein)</td>
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<tr>
<td></td>
<td>Oblice</td>
<td>1.170 ± 0.25</td>
<td>1.171 1.62 1.1835 1.22 1.1866 ± 0.07</td>
</tr>
<tr>
<td></td>
<td>Hanover</td>
<td>1.1094 0.08 0.08</td>
<td>1.1485 ± 0.02</td>
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<td>Kiel</td>
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<td>1.1485 ± 0.02</td>
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<tr>
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<td>1.1094 0.08 0.08</td>
<td>1.1485 ± 0.02</td>
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<td>Sandan-</td>
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<td>Troms-</td>
<td>1.1094 0.08 0.08</td>
<td>1.1485 ± 0.02</td>
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<td>Vasa</td>
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<td></td>
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<tr>
<td></td>
<td>Vasa</td>
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</table>

**Tab. 2:** Regional tidal parameters for the main waves in North and Central Europe

(= amplitude factor, = phase difference), compiled from literature.
<table>
<thead>
<tr>
<th>Instrument LCR</th>
<th>epoch</th>
<th>gravity range</th>
<th>number of observed $\Delta g$</th>
<th>calibration terms and r.m.s.e.</th>
<th>significant change of the calibration terms and cause</th>
<th>calibration terms and r.m.s.e.</th>
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<td>quadratic factors</td>
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<td>3</td>
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<td>5</td>
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<td>7</td>
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<td>4</td>
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<td>$\pm$ 9</td>
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</tr>
<tr>
<td></td>
<td>1971</td>
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<td>8</td>
<td>1.00043</td>
<td>$\pm$ 10</td>
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</tr>
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<tr>
<td></td>
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<td>22</td>
<td>1.00058</td>
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<tr>
<td></td>
<td>1978</td>
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<td>12</td>
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<td>$\pm$ 2</td>
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</tr>
<tr>
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<tr>
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<td>0.99990</td>
<td>$\pm$ 2</td>
<td>no</td>
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<td>1976</td>
<td>Paris-Hamburg</td>
<td>3</td>
<td>0.99990</td>
<td>$\pm$ 11</td>
<td>no, but repair work in dec. 1975</td>
</tr>
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<td>12</td>
<td>0.99994</td>
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<td>no</td>
</tr>
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<td></td>
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<td>Hannover-Hammerfest</td>
<td>22</td>
<td>1.00206</td>
<td>$\pm$ 6</td>
<td>yes, because of repair work in dec. 1978 and different gravity range</td>
</tr>
<tr>
<td></td>
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<td></td>
<td>0.99772</td>
<td>0.000000042 $\pm$ 18 $\pm$ 4</td>
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<tr>
<td>G 87</td>
<td>1975</td>
<td>Hannover-Bodø</td>
<td>6</td>
<td>1.000028</td>
<td>$\pm$ 3</td>
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</tr>
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<td>G 298</td>
<td>1976</td>
<td>Paris-Kopenhagen</td>
<td>22</td>
<td>1.00072</td>
<td>$\pm$ 3</td>
<td></td>
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<td></td>
<td>1978</td>
<td>München-Hamburg</td>
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<td>1.00073</td>
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<td></td>
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<td>Hannover-Hammerfest</td>
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<td>1.00083</td>
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</tr>
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<td></td>
<td>1980</td>
<td>Hannover-Hammerfest</td>
<td>25</td>
<td>1.00083</td>
<td>$\pm$ 2</td>
<td>no</td>
</tr>
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Tab. 3: Adjusted linear and quadratic calibration terms
### Tab. 4: Variance-covariance matrix of the station gravity values in μGal² (upper right of symmetric matrix)

### Tab. 5: Periodic calibration terms of LCR model G and D — gravity meters and calibration lines used
<table>
<thead>
<tr>
<th>Calibration line</th>
<th>Instrument</th>
<th>Epoch</th>
<th>Reading system</th>
<th>Number of observed I(g)</th>
<th>R.m.s. error of one I(g) ((\mu)Gal)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
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<td></td>
<td>Linear and quadratic scale factors</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Linear and periodic terms</td>
<td></td>
</tr>
<tr>
<td>Vertical</td>
<td>G 79</td>
<td>1979</td>
<td>Optical</td>
<td>110</td>
<td>+ 8</td>
</tr>
<tr>
<td>Calibration line</td>
<td>G 79</td>
<td>1981</td>
<td>Dig.voltm.</td>
<td>71</td>
<td>+ 4</td>
</tr>
<tr>
<td>Hannover</td>
<td>G 85 (1.1.79)</td>
<td>1979</td>
<td>&quot;</td>
<td>101</td>
<td>+ 5</td>
</tr>
<tr>
<td>(0.2 and 2 mGal)</td>
<td>G 298</td>
<td>1977</td>
<td>&quot;</td>
<td>110</td>
<td>+ 5</td>
</tr>
<tr>
<td></td>
<td>D 14</td>
<td>1976</td>
<td>Galvanometer</td>
<td>80</td>
<td>+ 5</td>
</tr>
<tr>
<td></td>
<td>D 14</td>
<td>1977/1981</td>
<td>Dig.voltm.</td>
<td>213</td>
<td>+ 5</td>
</tr>
<tr>
<td></td>
<td>D 23</td>
<td>1981</td>
<td>&quot;</td>
<td>109</td>
<td>+ 3</td>
</tr>
</tbody>
</table>

\( \Sigma = 794 \)

| Vertical         | G 79       | 1979   | Optical        | 220                         | + 8                             | + 10                          | + 8 |
| Calibration line | G 79 (1.1.80) | 1981 | Dig.voltm.     | 138                         | + 6                             | + 7                           | + 6 |
| Hannover         | G 85 (1.1.79) | 1979 | "              | 219                         | + 5                             | + 7                           | + 6 |
| (21 mGal)        | G 298      | 1977   | "              | 213                         | + 5                             | + 8                           | + 6 |
|                  | D 14       | 1976   | Galvanometer   | 114                         | + 5                             | + 6                           | + 5 |
|                  | D 14       | 1977   | Dig.voltm.     | 220                         | + 5                             | + 8                           | + 5 |
|                  | D 23       | 1981   | "              | 42                          | + 5                             | + 8                           | + 5 |

\( \Sigma = 1166 \)

<table>
<thead>
<tr>
<th>Calibration line</th>
<th>Instrument</th>
<th>Epoch</th>
<th>Reading system</th>
<th>Number of observed I(g)</th>
<th>R.m.s. error of one I(g) ((\mu)Gal)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
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<td>Linear and quadratic scale factors</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
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<td></td>
<td>Linear and periodic terms</td>
<td></td>
</tr>
<tr>
<td>Hanover - Harz</td>
<td>G 79</td>
<td>1978/79</td>
<td>Optical</td>
<td>279</td>
<td>+ 11</td>
</tr>
<tr>
<td>(190 mGal)</td>
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<td>1981</td>
<td>Dig.voltm.</td>
<td>99</td>
<td>+ 8</td>
</tr>
<tr>
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<td>1978</td>
<td>Optical</td>
<td>170</td>
<td>+ 10</td>
</tr>
<tr>
<td></td>
<td>G 85 (1.1.79)</td>
<td>1979</td>
<td>Dig. voltm.</td>
<td>63</td>
<td>+ 11</td>
</tr>
<tr>
<td></td>
<td>G 249</td>
<td>1978</td>
<td>Optical</td>
<td>43</td>
<td>+ 11</td>
</tr>
<tr>
<td></td>
<td>G 298</td>
<td>1977/1980</td>
<td>Dig.voltm.</td>
<td>388</td>
<td>+ 13</td>
</tr>
<tr>
<td></td>
<td>G 432</td>
<td>1978</td>
<td>&quot;</td>
<td>45</td>
<td>+ 15</td>
</tr>
<tr>
<td></td>
<td>G 562</td>
<td>1981</td>
<td>&quot;</td>
<td>98</td>
<td>+ 12</td>
</tr>
<tr>
<td></td>
<td>D 14</td>
<td>1977/1981</td>
<td>&quot;</td>
<td>493</td>
<td>+ 12</td>
</tr>
<tr>
<td></td>
<td>D 23</td>
<td>1978/1979</td>
<td>&quot;</td>
<td>337</td>
<td>+ 12</td>
</tr>
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</table>

\( \Sigma = 2015 \)

| European         | G 79       | 1978/1979 | Optical        | 38                          | + 13                            | + 14                          | + 13 |
| Calibration line | G 79 (1.1.79) | 1979 | Dig.voltm.     | 22                          | + 12                            | + 20                          | + 13 |
|                  | G 249      | 1978/1979 | Optical        | 30                          | + 14                            | + 21                          | + 16 |
|                  | G 298      | 1978/1979 | Dig.voltm.     | 32                          | + 15                            | + 21                          | + 16 |
| (1900 mGal)      | D 14       | 1978/1979 | "              | 24                          | + 15                            | + 15*                         | + 16 |

\( \Sigma = 157 \)

Tab. 5: Observations on the calibration lines and accuracies obtained

*) A linear scale factor has been introduced for each reset position
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<th>period</th>
<th>amplitude (uGal)</th>
<th>phase (°)</th>
</tr>
</thead>
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<td>(counter unit, appr. 1 mGal for model G and (\frac{1}{10}) mGal for model D)</td>
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<td></td>
</tr>
<tr>
<td>G 79</td>
<td>1.00</td>
<td>3.0 ±0.5</td>
<td>294 ±10</td>
</tr>
<tr>
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<td>7.88</td>
<td>7.4 ±0.5</td>
<td>123 ±4</td>
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<tr>
<td></td>
<td>35.47</td>
<td>5.9 ±0.6</td>
<td>156 ±7</td>
</tr>
<tr>
<td></td>
<td>70.94</td>
<td>9.2 ±0.8</td>
<td>192 ±6</td>
</tr>
<tr>
<td></td>
<td>1206.00</td>
<td>19.0 ±5.2</td>
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<td>4.4 ±0.5</td>
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<td>8.1 ±0.5</td>
<td>357 ±4</td>
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<td>35.47</td>
<td>7.3 ±1.1</td>
<td>358 ±8</td>
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<tr>
<td></td>
<td>70.94</td>
<td>11.1 ±1.4</td>
<td>119 ±7</td>
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<td>2.7 ±1.1</td>
<td>240 ±23</td>
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<tr>
<td></td>
<td>70.94</td>
<td>19.2 ±1.3</td>
<td>163 ±3</td>
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<tr>
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<td>3.7 ±0.5</td>
<td>239 ±7</td>
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<tr>
<td></td>
<td>3.94</td>
<td>0.8 ±0.4</td>
<td>346 ±28</td>
</tr>
<tr>
<td></td>
<td>7.88</td>
<td>6.8 ±0.5</td>
<td>208 ±4</td>
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<tr>
<td></td>
<td>70.94</td>
<td>14.0 ±1.7</td>
<td>159 ±7</td>
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<td>8.3 ±3.2</td>
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<td></td>
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<td>9.8 ±3.4</td>
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<td>1.00</td>
<td>0.8 ±0.4</td>
<td>93 ±29</td>
</tr>
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<td>3.94</td>
<td>1.3 ±0.4</td>
<td>94 ±18</td>
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<td>3.8 ±0.8</td>
<td>108 ±11</td>
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<td>70.94</td>
<td>6.6 ±0.8</td>
<td>244 ±8</td>
</tr>
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<td>35.47</td>
<td>4.0 ±2.0</td>
<td>277 ±29</td>
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<td>7.9 ±2.0</td>
<td>311 ±14</td>
</tr>
<tr>
<td></td>
<td>603.00</td>
<td>20.0 ±6.5</td>
<td>208 ±14</td>
</tr>
<tr>
<td>G 562</td>
<td>35.47</td>
<td>4.3 ±0.9</td>
<td>356 ±15</td>
</tr>
<tr>
<td></td>
<td>70.94</td>
<td>6.6 ±1.2</td>
<td>245 ±10</td>
</tr>
<tr>
<td>D 14</td>
<td>7.22</td>
<td>0.6 ±0.3</td>
<td>202 ±31</td>
</tr>
<tr>
<td></td>
<td>16.25</td>
<td>1.4 ±0.3</td>
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</tr>
<tr>
<td>D 23</td>
<td>32.5</td>
<td>3.0 ±0.5</td>
<td>238 ±7</td>
</tr>
</tbody>
</table>

Tab. 7: Periodic calibration terms, computed using 4100 observed gravity differences
A MODEL FOR STUDYING TEMPERATURE EFFECTS ON LACOSTE-ROMBERG GRAVIMETERS

by C. Gerstenecker,
Institut für Physikalische Geodäsie
Technische Hochschule, Darmstadt

ABSTRACT

According to the rules of control theory a model of the heating system of LaCoste-Romberg field gravimeters is developed. Such a model shows the connection between the temperature induced drift of the gravimeter and the temperature changes via the convolution integral. A first estimation of the eigenperiod, dead-time and time constant of the heating system is given. Experiments to improve these values and to determine the unknown parts of the transfer function are proposed.

1. INTRODUCTION

Gravimeters used for high precision gravity measurements are commonly thermostatized to minimize environmental temperature changes. Nevertheless, temperature changes induce an irregular "temperature drift", which is hardly to avoid. Attempts to eliminate the temperature drift by means of experimentally determined linear regression coefficients and temperature measurements are not successful. Results valid for general conditions couldn't be obtained.

The development of automatic data registration systems and process calculators suggests the use of more sophisticated models which are based on the methods of control theory. Such a model is presented in the next following 3 sections, whereby in the first part the used terminology and the model are explained, in the second our knowledge about the different functions of the model is discussed and in the last section numerical values and first results obtained from earlier experiments are given; experimental arrangements are proposed to get a better description of the model.

2. DESCRIPTION OF THE HEATING SYSTEM

In the terminology of the control theory the heating system of the LaCoste Romberg field gravimeter is a control process of the "on-off" type (P. Davies, 1965, p. 166).
The system consists of the gravimeter box containing the gravimeter spring, the heating source and a thermistor as the temperature sensing device (Fig. 1). The thermistor is regulating a switch, to switch on or off the heating source depending on the temperature \( b(t) \) inside the gravimeter. \( b(t) \) is changing around the preliminary set temperature \( r(t) = \Theta_s \) depending on the sensitivity range of the thermistor \( \Delta \Theta \) and the deadtime \( \tau \) of the heating system.

The sensitivity range can be assumed according to specifications of the manufacturers to be within 0.002° C, while the dead time \( \tau \) depends on the temperature gradient between heat source and thermistor and on the resistance of the heat transfer path (P.Davies, 1965, p. 168).

The feedback system is disturbed by the outer temperature \( u(t) \), causing additional changes of \( b(t) \) and the gravimeter output \( y(t) \).

The block diagram of the system is shown in Fig. 2. If we assume that \( G_1 \) and \( G_2 \) are linear processes following a linear differential equation as

\[
\sum_{i=0}^{n} a_i \frac{d^i y}{dt^i} = \sum_{i=0}^{m} b_i \frac{d^i x}{dt^i} \tag{1}
\]

the transfer function of the system \( p(t) \) can be obtained by the superposition theorem (Di Stefano III, et al., 1976, p. 31). To do this the Laplace-transformation of the input- and output function has to be performed. In Table 1 all used functions and their meaning in the time- and frequency domain are listed.

<table>
<thead>
<tr>
<th>Function</th>
<th>Time domain</th>
<th>Frequency domain</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>Input</td>
<td>( r(t) )</td>
<td>( R(s) )</td>
<td>setted temperature</td>
</tr>
<tr>
<td></td>
<td>( u(t) )</td>
<td>( U(s) )</td>
<td>environmental temperature</td>
</tr>
<tr>
<td>Transfer</td>
<td>( p(t) )</td>
<td>( P(s) )</td>
<td>Transfer of the whole system</td>
</tr>
<tr>
<td></td>
<td>( p_R(t) )</td>
<td>( P_R(s) )</td>
<td>( \text{T-f if } u(t) = 0 )</td>
</tr>
<tr>
<td></td>
<td>( y(t) )</td>
<td>( Y(s) )</td>
<td>( \text{T-f if } r(t) = 0 )</td>
</tr>
<tr>
<td></td>
<td>( y_R(t) )</td>
<td>( Y_R(s) )</td>
<td>temperature induced drift</td>
</tr>
<tr>
<td></td>
<td>( y_U(t) )</td>
<td>( Y_U(s) )</td>
<td>( &quot; &quot; u(t) = 0 )</td>
</tr>
</tbody>
</table>

Reducing the block diagram to its canonical form (Di Stefano III et al., 1976, p. 117) the output function \( Y(s) \) can be expressed by the input function \( U(s) \) and \( R(s) \) and the transfer functions \( P_R(s) \) and
Fig. 1: Model of the Heating System

Fig. 2: Blockdiagram

Fig. 3: Diagram of temperature $\theta(t)$ inside the box
\( P_Y(s) \). Particularly we obtain

\[
Y_R(s) = \frac{G_1 \cdot G_2}{1 + G_1} R(s) \quad (2) \quad \text{for} \ u(t) = 0,
\]

\[
Y_U(s) = \frac{G_2}{1 + G_1} U(s) \quad (3) \quad \text{for} \ r(t) = 0
\]

and by using the superposition theorem

\[
Y(s) = Y_R(s) + Y_U(s) = \frac{G_2(G_1 \cdot R(s) \cdot U(s))}{1 + G_1}
\]  

(4)

In the time domain the output function \( y(t) \) is obtained as the convolution integral of the transfer-(weight)-function and the input function \( u(t) \)

\[
y(t) = \int_0^{+\infty} p(t-\tau) \cdot u(\tau) \cdot d\tau \quad (5)
\]

or by the inverse Laplace transformation \( L^{-1} \) of \( Y(s) \)

\[
y(t) = L^{-1}(Y(s)) \quad .
\]  

(6)

Equations (4) and (5) demonstrate that the temperature induced drift at the time \( t \) depends not only on the environmental temperature \( u(t) \) at \( t \), but on the whole "history" of \( u(t) \), whereby \( t \) is running from 0 to infinity.

3. DISCUSSION OF THE MODEL

The model can be discussed by looking at the different functions. In the first state special informations can be obtained, about what we have to observe to get better instrument dependent constants of the model.

3.1 Input functions

The input function \( r(t) \) is the preliminary setted temperature \( \theta_s \) to which the space inside the gravimeter box shall be heated. Therefore, we can write

\[
r(t) = \text{const} = \theta_s \quad (7a)
\]

or

\[
R(s) = \theta_s \cdot \frac{1}{s} \quad .
\]  

(7b)

\( u(t) \) is the environmental temperature which is disturbing the temperature \( \theta_s \) inside the gravimeter box and generates the actual temperature \( \theta(t) \) (Fig.2). \( u(t) \) is not predictable and has to be mea-
sured therefore, using mechanical or electronic thermometers during the whole time, where the instrument is heated.

3.2 Transfer function

The transfer function depends as shown in equation (4) on the controller G1 and the process G2. While G1 is the heat source, the process G2 has to be understood as the conversion of temperature changes due to the temperature induced gravimeter drift. Some information about both blocks can be obtained, looking at each of them separately.

3.2.1 The transfer function of a regulated heating system without the process G2 and the noise input u(t) is given by

\[ P_H(s) = \frac{G1}{1 + G1} \]  \hfill (8)

If we assume that each point inside the box has the same temperature \( \theta \) and the heat exchange with the space outside the box is proportional to the relation of the two temperatures inside the box the heating system can be described by (Magnus, 1976, p.118)

\[ C \cdot \frac{d \theta}{dt} + k \cdot \theta = N \]  \hfill (9)

whereby

- \( C \) = heat capacity
- \( k \) = proportionality constant
- \( N \) = power of the heating source/sec.

Introducing new symbols as

\( C/k = T_{G1} \) = time constant of the simplified heating system

\( N/k = \theta_s \) = setted temperature

(9) is changed to

\[ T_{G1} \dot{\theta} + \theta = \theta_s \]  \hfill (10)

The solution of (10) is

\[ \theta(t) = \theta_s - (\theta_s - \theta_A) \exp\left(-t/T_{G1}\right). \]  \hfill (11)

The transient solution of \( \theta(t) \) is not unique and depends on the starting conditions

\[ \theta_A < \theta_s \]  \hfill (12)
while the steady state response converges to $\theta_s$ (Fig. 3).

Equation (11) is valid only if no temperature gradient exists between heat source and thermistor. Really the temperature $\theta_2$ generated by the heat source needs the time interval $\tau$ (dead time) to be transferred to the thermistor. The range within the temperature will change inside the box is according (Davies, 1965, p. 172)

$$\theta_2 - \theta_s = -(1 - e^{-t/T_{G1}}) \theta_s \cdot T_{G1}$$  \hspace{1cm} (13)

showing, that the biggest possible error $(\theta_2 - \theta_s)$ is equal $-\theta_s T_{G1}$.

Finally we look at the Laplace transform $\Theta(s)$ of $\theta(t)$ (equation (10)), which we obtain as (DiStefano III et al., 1976, p. 60)

$$\Theta(s) = \frac{\theta_s + T_{G1} \theta_A \cdot s}{s(T_{G1} \theta_A \cdot s + 1)} .$$ \hspace{1cm} (14)

There the input function $r(t) = \theta_s$ vice versa

$$P(s) = \frac{\theta_s \cdot \frac{1}{s}}{s}$$ \hspace{1cm} (15)

was introduced.

The transfer function $P_{R}(s)$ can be calculated from (8) and (14)

$$P_{R}(s) = \frac{\Theta(s)}{R(s)} = \frac{T_{G1}}{1 + T_{G1} s} = \frac{1}{1 + T_{G1} s}$$ \hspace{1cm} (16)

Solving (16) for $G_1(s)$ is obtained

$$G_1(s) = \frac{1}{T_{G1} s} .$$ \hspace{1cm} (17)

3.2.2 Informations about the process $G_2$ are hardly to get. The simplest model would be to use the linear temperature coefficient of the spring material, but the experience confirms that such models do not work satisfying. More successful seems to be the development of a temperature dependent rheological model. To do this some informations should be available about $G_2$ from experiments, where the input $u(t)$ and the output $y(t)$ have to be observed.

3.3 Output function

The desired output function $y(t)$ is

$$y(t) = \text{const}$$ \hspace{1cm} (18)

Stable feedback systems realizing (18), however do not exist. The
The most common physical realization of \( y(t) \) is a second order linear differential equation as

\[
\frac{d^2 y}{dt^2} + 2 \zeta \omega_0 \frac{dy}{dt} + \omega_0^2 y = \omega_0^2 x.
\]  

Since \( y(t) \) is observable, the damping factor \( \zeta \) and the natural angular frequency \( \omega_0 \) can be determined by a least square fit.

From (4) and (17) \( G_2 \) is computed according

\[
G_2 = \frac{Y(s) (T_{Gl} s + 1)}{1 + U \cdot T_{Gl} \cdot s^2}.
\]

4. NUMERICAL VALUES

For the LaCoste Romberg gravimeter G-258 some typical available values of the heating system are given in Table 2.

**Table 2:** Values for the heating system of the G-258

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Settled temperature</td>
<td>( \theta_s )</td>
<td>52° C</td>
</tr>
<tr>
<td>Sensitivity of the thermistor</td>
<td>( \Delta \theta )</td>
<td>0.002° C</td>
</tr>
<tr>
<td>dead time</td>
<td>( T )</td>
<td>10 sec</td>
</tr>
<tr>
<td>power/sec</td>
<td>( N )</td>
<td>12 watt/sec</td>
</tr>
<tr>
<td>proportionality factor</td>
<td>( k )</td>
<td>0.23</td>
</tr>
<tr>
<td>damping factor</td>
<td>( \zeta )</td>
<td>30 minutes</td>
</tr>
<tr>
<td>Eigen period</td>
<td>( \omega_0 )</td>
<td>0.00175 C/sec</td>
</tr>
<tr>
<td>natural angular frequency</td>
<td></td>
<td>1145.9</td>
</tr>
</tbody>
</table>

The values for \( \zeta \) and \( \omega_0 \) are estimated from experiments published by Gerstenecker (1977).

The estimation of \( T_{Gl} \) is only correct if the assumption that the time constant \( T_y(t) \) of the output function \( y(t) \)

\[
T_{Gl} = T_y(t) = \frac{1}{\zeta \omega_0}
\]

is valid, what should be shown by measurements.

5. CONCLUSIONS

As shown in section 2 and 3 the temperature induced drift \( Y(s) \) is connected to outside temperature changes \( U(s) \) in the frequency
domain via the transfer function $P(s)$.

$P(s)$ is composed of the functions $G_1$ and $G_2$ whereby $G_1$ can be specified by using instructions of the manufacturer and simple observations.

Experiments have to be carried out to find an appropriate model of the process $G_2$. In such experiments the disturbing outside temperature $u(t)$ and the temperature induced drift have to be observed. Since $y(t)$ is calculated from $u(t)$ by means of convolution with the transfer function $p(t)$, $u(t)$ must be measured additionally during the transport and rest time of gravimeter measurements. The data collection and computation should be carried out with digital data acquisition systems and online computers.

Hitherto available experimental results demonstrate above all the sensitivity of the gravimeters against outside temperature changes. Especially temperature jumps generated by unpacking the instrument from the transportation box cause irregular drifts of 5 and more microgals. Particularly periodic temperature changes with periods near 30 minutes producing an unstable drift curve have to be avoided.

6. REFERENCES


Magnus, K.: Schwingungen, Stuttgart 1976
RESULTS OF CIRCULAR ERROR STUDIES WITH LACOSTE AND ROMBERG
GRAVITY METERS

Matthias Becker
Institut für Physikalische Geodäsie
Technische Hochschule Darmstadt

Abstract:
Starting with a look at the LaCoste measuring system as used both in the model G and model D gravimeter an investigation of the deficiency in calibration of the D-38 gravimeter is performed. Periodic errors are determined on a specially designed calibration line using the reset-screw. Measuring a gravity difference of about 40 mgal the change of the calibration factor at different parts of the meter range is studied. In addition the high frequency periodic errors of the G-258 gravimeter are determined.

Contents

1. Introduction
2. Technical details about the measuring system
   2.1 Periodic errors
3. The calibration line and the "self calibration"
4. The mathematical model for adjusting the observations
5. Observations and results
   5.1 Periodic errors of D-38
   5.2 Periodic errors of G-258
6. Measurements to check the non-linearities of the scale factor
7. Conclusions

Appendix: Hypothesis testing in the general linear model
1. Introduction

The manufacturer claims that for the model D-microgravimeter the periodic errors can be neglected. The increase of the reduction ratio of the lever-system should have reduced them below the detectable level. Just as in the model G-meter the most important periods stem from the turn of the measuring screw. These periods are 70.94 and 35.47 C.U.\(^*\) for the G-meter. In measuring small gravity differences, such as zero lines and zero-networks their effect can be reduced and often neglected. However in the model J-meter the most critical screw-periods are 3.25 and 1.625 C.U. (in order to allow a comparison with the G-meter we chose the same Units, which means reading times \(10 \exp -2\) or approximate milligals), which are well within the range of zero nets. This paper deals with the determination of the amplitude and the phase lag of this error in the D-38 gravity meter. Using the reset-mechanism a 'self-calibration' on a special designed calibration line was performed. Additionally the change of the scale-factor with different reset-positions was investigated.

2. Technical details about the measuring system

Prior to our investigations we found it useful to take a look at the construction of the model D and model G gravimeter. The main difference of the two instruments stems from the reduced range of the D-meter and the higher resolution so obtained. This was reached by changing the gear-box-ratio and by increasing the reduction ratio of the lever system (Harrison, LaCoste, 1978). The following values are valid for the two types of instruments.

\(^*\) C.U. = Counter Units
1 turn of the spindle corresponds to [C.U.]

<table>
<thead>
<tr>
<th></th>
<th>G</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>gear box ratio</td>
<td>70.94:1</td>
<td>32.5:1</td>
</tr>
<tr>
<td>pitch of the spindle</td>
<td>184</td>
<td>184</td>
</tr>
<tr>
<td>reduction ratio of the lever system</td>
<td>77.8:1</td>
<td>1698:1</td>
</tr>
<tr>
<td>positioning accuracy of the support corresponding to 1 μgal</td>
<td>2.5 (10^{-9}) cm</td>
<td>2.5 (10^{-9}) cm</td>
</tr>
<tr>
<td>required accuracy of the spindle to achieve 1 μgal accuracy</td>
<td>1.95 (10^{-7}) cm</td>
<td>4.25 (10^{-6}) cm</td>
</tr>
<tr>
<td>mean amplitude of periodic error</td>
<td>35 μgal</td>
<td>1 μgal?</td>
</tr>
<tr>
<td>resulting real deviations in the measuring screw</td>
<td>6.8 (10^{-6}) cm</td>
<td>4.25 (10^{-6}) cm</td>
</tr>
</tbody>
</table>

Tab. 2.1

2.1. Periodic errors

As can be seen from table 2.1, there is at least a mean periodic error in the D-meter with an amplitude of 1.6 μgal, assuming the calibration factor equal to one. Below are the toothed-wheels as mounted in the gear boxes of the two gravimeters.

![Diagram of Model G and Model D](image)

Fig. 2.1.
From these values the following periods can be expected.

<table>
<thead>
<tr>
<th>G</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>1206.0</td>
<td>6.50</td>
</tr>
<tr>
<td>603.0</td>
<td>3.25</td>
</tr>
<tr>
<td>70.94</td>
<td>1.625</td>
</tr>
<tr>
<td>35.47</td>
<td>0.72</td>
</tr>
<tr>
<td>7.89</td>
<td>0.361</td>
</tr>
<tr>
<td>3.94</td>
<td>0.1</td>
</tr>
<tr>
<td>1.00</td>
<td></td>
</tr>
</tbody>
</table>

Tab. 2.2. Periodic errors to be expected in the LCR gravimeters [C.U.=approx. milligals]

The most critical periods are those of once and twice per turn of the spindle. This leads to periods of 70.94 and 35.47 C.U. in the G-meter. The mean amplitude for this error is taken from (Harrison, LaCoste, 1978). As far as the author knows there is no investigation of the measuring screws themselves, where they are checked for periodic and linear errors. One might think of a individual calibration table for each screw. Till now the only hints on their precision can be found in some research done on the tilt table of the Radian Corporation (Electro-Leves Division, 1973). In this instrument the same type spindle is used. The investigations of (Gerstenecker, 1978), (Harrison, 1976) indicate bigger deviations from the average pitch. Gerstenecker found values of up to 2.5 µm. Additionally there is the indication of local jumps in the pitch being bigger than the cyclic error. The crux of all these investigations is that the detected deviations are within the error bounds of the means used for the calibration of the tilt-tables. So it is quite doubtful whether these large values can be accepted for the gravity-meter spindles. Nevertheless they show the necessity of a detailed calibration of the measuring screw.
3. The calibration line and the 'self-calibration'

The ideal situation for the determination of periodic errors would be a calibration line which is determined independently from any measuring screw error, e.g. by an instrument with an electronic feedback system. As long as such well determined points are not available there are two possibilities. The first one is the use of several different LaCoste and Romberg gravity meters in many independent observation series. In this way one can hope to get a good determination of the line's gravity values (Gerstenecker, 1973). In the adjustment following one can determine both the unknown gravity values and the periodic errors for each instrument. The periodic errors are detectable because of the inter-instrument differences.

The other way is to use only one instrument. A calibration line is then reobserved in some interval of time. Under the assumption of no secular change in gravity and choosing the intervals according to the meter drift one can get a coverage of the periods to be detected. In this case the periodic errors have a different effect on identical gravity differences and can so be determined in an adjustment. Moreover they are not affected by the other systematic errors different gravimeters may have.

Using the reset mechanism in the model D gravimeter the same situation can be attained. The gravity differences in the calibration line are about 0.85 mgal which corresponds to a $94^\circ$ turn of the measuring screw. Between each series of observations we changed the reset by 1.4 mgal, which is the smallest amount to be read from the reset-counter. With six measurement series we got an almost complete coverage of the measuring screw (see Fig. 3.1.). In this manner the calibration was performed without employing any other instruments as reference.

---

Fig. 3.1. Distribution of the reading position for one gravity difference over the measuring screw
Fig. 3.2. Staircase calibration line
Subdivision 0.2 and 0.8 mgal
The calibration line has a maximum gravity difference of 7.5 mgal. This led to 10 points with a mean gravity difference of 0.85 mgal (see Fig. 3.2.). All periods expected for the D-meter are contained at least once. The number of points was limited to 10 in order to render possible three repeated measurements at every point. Every measuring series had 30 observations and was performed in a ladder sequence. In order to determine the high frequency periods of our model G gravimeter the lower part of the line was further subdivided into 0.2 mgal intervals with a maximum range of 4.0 mgal. The points are situated on a steel concrete staircase. To prevent disturbances of the readings by the nearby elevator or by the public, the observations took place only at nighttime or on weekends when the building was closed.

4. The mathematical model for adjusting the observations

Starting values for the adjustments were the corrected gravimeter readings. The following corrections were applied.

1. Conversion to approximate milligal-values using the manufacturer's table.*

2. Height correction

3. Air-mass correction

4. Tidal corrections. The theoretical tides are computed with the Cartwright-Taylor-Edden tidal potential development. They are fitted to the Darmstadt area using the results of the Darmstadt earth tide measurements. (Gerstenecker, Groten, 1976).

The observation equation for one instrument reads

\[ V_i = G_j - R_i + O_k + D_{K,1}(t) + A_n \sin(Z_i^1 + B_n) + K_m(Z_i) \]

\[ V_i = \text{Residual of observation no. } i \]

\[ G_j = \text{Gravity value of point } j \]

\[ R_i = \text{Corrected gravimeter reading} \]

*) An additional scale factor for the conversion to the IGSN-scale is used.
$O_K =$ Orientation unknown of series K
$D_{K,t} =$ Drift polynomial of series K, $l =$ order of drift-polyonomial, $t =$ time
$A_n =$ Amplitude of periodic error $n$
$B_n =$ Phase lag $n$
$Z_i =$ $Z_i \cdot 2\pi \cdot T_n$
$K_m =$ Arbitrary function for the non-periodic calibration errors, in general a polynomial of degree $m = 1$

The model is designed to allow the introduction of a Serbetci type correlation between the gravimeter readings (Serbetci, 1968). Furthermore an hypothesis testing algorithm is included (see appendix for a detailed description).

5. Observations and results

5.1. Periodic errors of the D-38

On the calibration line as described in chapter 3 six series of measurements with 30 observations each were performed. Because of the total gravity difference of only 7.4 mgal and the small reset-shift of about 0.12 turns the introduction of scale factors turned out to be unnecessary. The instrument had a positive parabolic drift throughout the measurements. The accuracy attained varied only about 0.8 µgal among all series (see Tab. 5.1.1.) therefore no weights were introduced to the common adjustment.

<table>
<thead>
<tr>
<th></th>
<th>Drift coefficients [$\mu$gal /h]</th>
<th>M.S.E. of gravity values [$\mu$gal]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>[$\mu$gal /h * $\sigma_2$]</td>
<td></td>
</tr>
<tr>
<td>D1</td>
<td>5.35±0.40</td>
<td>-0.78±0.31</td>
</tr>
<tr>
<td>D2</td>
<td>3.07±0.34</td>
<td>-1.17±0.27</td>
</tr>
<tr>
<td>D3</td>
<td>4.85±0.25</td>
<td>-1.38±0.19</td>
</tr>
<tr>
<td>D4</td>
<td>3.24±0.28</td>
<td>-0.46±0.21</td>
</tr>
<tr>
<td>D5</td>
<td>4.38±0.46</td>
<td>-1.01±0.42</td>
</tr>
<tr>
<td>D6</td>
<td>4.45±0.45</td>
<td>-1.10±0.45</td>
</tr>
</tbody>
</table>

Tab. 5.1.1.
Residuals
ADJ.PER.ERROS DRAWN
P211

Fig. 5.1.1.

Residuals
ADJ.PER.ERROS DRAWN
P210

Fig. 5.1.2.
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>-</td>
<td>4.01</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>1.625</td>
<td>3.35</td>
<td>2.98±0.36</td>
<td>145.97±7.03</td>
<td>33.93</td>
<td>0.0</td>
</tr>
<tr>
<td>3.25</td>
<td>3.55</td>
<td>2.49±0.38</td>
<td>48.28±8.70</td>
<td>21.97</td>
<td>0.0</td>
</tr>
<tr>
<td>6.5</td>
<td>3.96</td>
<td>1.04±0.42</td>
<td>196.59±23.83</td>
<td>2.92</td>
<td>5.71</td>
</tr>
<tr>
<td>1.625</td>
<td>2.75</td>
<td>3.01±0.30</td>
<td>146.53±5.71</td>
<td>44.10</td>
<td>0.0</td>
</tr>
<tr>
<td>3.25</td>
<td>2.53±0.29</td>
<td>49.33±6.64</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.625</td>
<td>2.65</td>
<td>3.02±0.29</td>
<td>146.28±5.47</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3.25</td>
<td>2.52±0.28</td>
<td>49.31±6.41</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6.5</td>
<td>1.04±0.28</td>
<td>194.97±15.60</td>
<td></td>
<td>33.95</td>
<td>0.0</td>
</tr>
</tbody>
</table>

Tab. 5.1.2. Results of the common adjustment. Testvariable is the ratio of the residual sum of squares (hypothesis)/residual sum of squares*, and the P-value is computed using Fisher’s F-distribution.

* see appendix
Tab. 5.1.2. gives the results of this adjustment. The gravity values of the 10 measuring points are determined with a mean square error of 0.9 µgal. The plot of the residuals shows clearly the systematic deviations caused by the periodic errors.

Fig. 5.1.1. is a plot of the residuals over the counter readings whereas Fig. 5.1.2. shows the residuals related to the measuring screw position in degrees. As can be seen there is no period besides these theoretically postulated. The periods of 1.625 and 3.25 corresponding to half and full turn of the screw are well determined both in phase and amplitude. The period 6.5 resulting from the combination of the gear-box wheels has an amplitude of only 1.04 µgal. Looking at the low significance level of 94.3% and the still remaining residuals this period must be regarded as not significant and can therefore be neglected.

5.2. Periodic errors of the G-258

The measurements took place on a part of the calibration line with 21 points and 0.2 mgal subdivision. The periodic errors were to be determined from the differences between 3 series measured with D-38 and 4 series measured with the G-258. Some of the series (1 of the D-38 and 3 of the G-258) were measured in two parts. The separate adjustment showed great discrepancies in the accuracy obtained (Tab. 5.2.1.). Therefore the measurements were weighted with \( P_1 = \text{const.} / \text{mgal}^2 \). Linear drift was eliminated in all cases.

The measurements of D-38 were corrected with the values found on the 8 mgal line. In the common adjustment we introduced the periods theoretically expected plus the period of 2.54. At this value the Fourier-analysis of the differences D-G obtained in the separate adjustments showed a significant peak. The results are listed in Tab. 5.2.2. For all periods, significant amplitudes were found, and especially the combination of the three periods led to unrealistically large values up to 8.4 µgal. This seemed to indicate that the reference measurements with the D-38 did not provide stable gravity values. Therefore a new adjustment including the measurements with D-38 on the 8 mgal range was performed and quite different results were obtained (see Tab. 5.2.3.).
<table>
<thead>
<tr>
<th>Nr.</th>
<th>drift [μgal/h]</th>
<th>M.S.E. of gravity values [μgal]</th>
<th>weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>D1</td>
<td>4.08±0.27</td>
<td>3.2</td>
<td>0.86</td>
</tr>
<tr>
<td>D2/1</td>
<td>5.80±0.95</td>
<td>6.9</td>
<td>0.19</td>
</tr>
<tr>
<td>D2/2</td>
<td>5.43±0.93</td>
<td>6.4</td>
<td>0.22</td>
</tr>
<tr>
<td>D3</td>
<td>5.56±0.26</td>
<td>2.4</td>
<td>1.53</td>
</tr>
<tr>
<td>G1/1</td>
<td>-0.61±0.24</td>
<td>3.1</td>
<td>0.92</td>
</tr>
<tr>
<td>G1/2</td>
<td>-1.15±0.38</td>
<td>4.1</td>
<td>0.54</td>
</tr>
<tr>
<td>G2/1</td>
<td>1.17±0.44</td>
<td>3.2</td>
<td>0.87</td>
</tr>
<tr>
<td>G2/2</td>
<td>1.68±0.54</td>
<td>3.8</td>
<td>0.62</td>
</tr>
<tr>
<td>G3/1</td>
<td>3.90±0.40</td>
<td>2.9</td>
<td>1.04</td>
</tr>
<tr>
<td>G3/2</td>
<td>0.76±0.61</td>
<td>4.7</td>
<td>0.41</td>
</tr>
<tr>
<td>G4</td>
<td>5.14±0.26</td>
<td>4.6</td>
<td>0.42</td>
</tr>
</tbody>
</table>

Tab. 5.2.1.

The values found for D-38 show only small differences from those shown in Tab. 5.1.2., being well within the range of the errors. Contrary to this, the situation for the G-258 changed completely. Considering the test variables and the levels of significance there is no error of period 2.54 and 3.94. Only the period of 1 mgal is stable and the amplitude of about 2.7 μgal is confirmed. These results show the sensitivity of the model to changes in the input data and to combinations of different periods and instruments.
<table>
<thead>
<tr>
<th>Period</th>
<th>MO [μgal]</th>
<th>Amplitude [μgal]</th>
<th>Phase [°]</th>
<th>Testvar.</th>
<th>P-value [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>-</td>
<td>4.29</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>1.00</td>
<td>4.21</td>
<td>2.46±0.73</td>
<td>224.86±17.55</td>
<td>5.72</td>
<td>0.37</td>
</tr>
<tr>
<td>2.54</td>
<td>4.10</td>
<td>3.98±0.79</td>
<td>103.01±11.47</td>
<td>12.87</td>
<td>0.01</td>
</tr>
<tr>
<td>3.94</td>
<td>4.17</td>
<td>4.62±1.14</td>
<td>137.01±10.01</td>
<td>8.52</td>
<td>0.03</td>
</tr>
<tr>
<td>1.00</td>
<td>4.01</td>
<td>2.64±0.70</td>
<td>213.94±15.75</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>2.54</td>
<td>4.08</td>
<td>4.15±0.78</td>
<td>108.15±10.81</td>
<td>10.27</td>
<td>0.0</td>
</tr>
<tr>
<td>1.00</td>
<td>3.78</td>
<td>2.55±0.71</td>
<td>225.44±16.42</td>
<td>7.70</td>
<td>0.01</td>
</tr>
<tr>
<td>3.94</td>
<td>3.94</td>
<td>4.76±1.12</td>
<td>138.34±9.45</td>
<td>19.53</td>
<td>0.0</td>
</tr>
<tr>
<td>2.54</td>
<td>3.94</td>
<td>6.95±0.95</td>
<td>92.82±7.59</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>1.00</td>
<td>3.65</td>
<td>8.15±1.18</td>
<td>169.60±7.03</td>
<td>17.35</td>
<td>0.0</td>
</tr>
<tr>
<td>2.54</td>
<td>3.94</td>
<td>2.91±0.65</td>
<td>208.46±12.98</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>3.94</td>
<td>3.94</td>
<td>7.45±0.80</td>
<td>98.00±6.78</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Tab. 5.2.2. Adjustment of G-258 and corrected D-38 measurements with 1 point fixed

<table>
<thead>
<tr>
<th>Period</th>
<th>MO [μgal]</th>
<th>Amplitude [μgal]</th>
<th>Phase [°]</th>
<th>Testvar.</th>
<th>P-value [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>-</td>
<td>4.25</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>D-1.625</td>
<td>3.45</td>
<td>3.13±0.28</td>
<td>145.84±5.15</td>
<td>55.17</td>
<td>0.0</td>
</tr>
<tr>
<td>D-3.25</td>
<td>3.35</td>
<td>2.79±0.29</td>
<td>52.87±5.69</td>
<td>59.02</td>
<td>0.0</td>
</tr>
<tr>
<td>D-1.625</td>
<td>3.35</td>
<td>3.17±0.27</td>
<td>145.35±4.95</td>
<td>12.94</td>
<td>0.0</td>
</tr>
<tr>
<td>D-3.25</td>
<td>3.35</td>
<td>2.79±0.28</td>
<td>51.93±5.55</td>
<td>53.51</td>
<td>0.0</td>
</tr>
<tr>
<td>G-1.00</td>
<td>3.35</td>
<td>2.73±0.54</td>
<td>227.58±11.96</td>
<td>55.62</td>
<td>0.0</td>
</tr>
<tr>
<td>D-1.625</td>
<td>3.44</td>
<td>3.00±0.28</td>
<td>146.60±5.46</td>
<td>2.69</td>
<td>6.88</td>
</tr>
<tr>
<td>D-3.25</td>
<td>3.44</td>
<td>2.86±0.29</td>
<td>55.59±5.75</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>G-2.54</td>
<td>3.44</td>
<td>1.44±0.62</td>
<td>94.64±25.55</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>D-1.625</td>
<td>3.44</td>
<td>3.14±0.28</td>
<td>146.09±5.13</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>D-3.25</td>
<td>3.44</td>
<td>2.90±0.30</td>
<td>55.44±5.82</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>G-3.94</td>
<td>3.35</td>
<td>1.49±0.85</td>
<td>2.16±27.73</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>D-1.625</td>
<td>3.35</td>
<td>3.04±0.28</td>
<td>144.20±5.32</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>D-3.25</td>
<td>3.35</td>
<td>2.84±0.29</td>
<td>53.43±5.82</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>G-1.00</td>
<td>3.35</td>
<td>2.82±0.55</td>
<td>255.16±11.81</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>G-2.54</td>
<td>3.35</td>
<td>1.85±0.85</td>
<td>129.12±61.63</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>G-3.94</td>
<td>3.35</td>
<td>1.04±0.88</td>
<td>273.71±54.48</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Tab. 5.2.3. Common adjustment of all observations
6. Measurements to check the nonlineairities of the scale factor

In order to investigate the variation in the scale factor with different reset-positions we installed two calibration ranges. The gravity differences are 42.20 and 38.20 mgal. The first one was observed in 12 series with 82 observations and 6 different positions. On the second one 6 series with 24 observations and again 6 reset positions took place. The reset was changed in steps of half a turn corresponding to about 30 mgal. In the adjustment we kept one position fixed and determined 5 scale factors. Unfortunately we did not know about the exact periods of the screw error at that time. It turned out that the gravity differences are non-integral multiples of the periods and we had to introduce the amplitudes and phases found in chapter 5. The results, both with and without the correction for periodic errors are listed in Tab. 6.1. and 6.2. The T-value in this case is the scale factor divided by its mean square error. The level of significance is given by the p-value according to (100-p)%.

There are variations in the scale factor of $7 \cdot 8 \cdot 10^{-4}$. The corrections cause significant changes up to $3.2 \cdot 10^{-4}$. Nevertheless the situation on the whole is not changed too much. As can be seen in Fig. 6.1. and 6.2. both independent investigations show a decrease of the scale factor with increasing dial readings. Somewhere between the counter positions 100 and 140 there is a jump in the factor and than the decrease continues. The results correspond in their magnitude with those published by (Lambert et al., 1979) and (Torge, Kambergieser, 1979).

The nonlinearities of scale in the G-meter are $1 \cdot 10^{-3}$ of full scale, with a smooth variation. This variation leads to differences between adjacent 100 mgal interval factors of up to $2 \cdot 10^{-4}$. Considering these values and taking into account the increased reduction ratio the variations in scale of the D-models might be caused by the lever system. Nevertheless the investigations of Torge and Lambert cited above speak for a reset-dependence of the scale factor, whereas in the most recent work of (Dragert et al., 1981) the changes in reduction factor of the lever system are determined by the LaCoste weight calibration apparatus. Dial factor curves similar to those of the G-meters are derived for 4 D-meters.
<table>
<thead>
<tr>
<th>Reset</th>
<th>Scale factor $[\times 10^{-4}]$</th>
<th>M.S.E. $[\times 10^{-4}]$</th>
<th>T-value</th>
<th>P-value $[^\circ]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. without correction $\Delta g = 42.1951 \pm 0.0014$ $MO = 0.0019$</td>
<td>51.0</td>
<td>4.052</td>
<td>0.716</td>
<td>5.6</td>
</tr>
<tr>
<td>50.5</td>
<td>1.876</td>
<td>0.741</td>
<td>2.5</td>
<td>1.54</td>
</tr>
<tr>
<td>50.0</td>
<td>0.000</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>49.5</td>
<td>2.992</td>
<td>0.850</td>
<td>3.5</td>
<td>0.11</td>
</tr>
<tr>
<td>49.0</td>
<td>1.220</td>
<td>0.663</td>
<td>1.8</td>
<td>7.32</td>
</tr>
<tr>
<td>48.5</td>
<td>-1.999</td>
<td>0.702</td>
<td>2.8</td>
<td>0.69</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Reset</th>
<th>Scale factor $[\times 10^{-4}]$</th>
<th>M.S.E. $[\times 10^{-4}]$</th>
<th>T-value</th>
<th>P-value $[^\circ]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2. corrected for periodic errors $\Delta g = 42.2011 \pm 0.0014$ $MO = 0.0019$</td>
<td>51.0</td>
<td>7.095</td>
<td>0.738</td>
<td>9.6</td>
</tr>
<tr>
<td>50.5</td>
<td>4.534</td>
<td>0.764</td>
<td>5.9</td>
<td>0.00</td>
</tr>
<tr>
<td>50.0</td>
<td>0.000</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>49.5</td>
<td>4.872</td>
<td>0.876</td>
<td>5.5</td>
<td>0.00</td>
</tr>
<tr>
<td>49.0</td>
<td>3.289</td>
<td>0.684</td>
<td>4.8</td>
<td>0.00</td>
</tr>
<tr>
<td>48.5</td>
<td>1.208</td>
<td>0.724</td>
<td>1.2</td>
<td>10.3</td>
</tr>
</tbody>
</table>

Tab. 6.1.

<table>
<thead>
<tr>
<th>Reset</th>
<th>Scale factor $[\times 10^{-4}]$</th>
<th>M.S.E. $[\times 10^{-4}]$</th>
<th>T-value</th>
<th>P-value $[^\circ]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. without correction $\Delta g = 38.1988 \pm 0.0036$ $MO = 0.0032$</td>
<td>51.1</td>
<td>5.908</td>
<td>0.167</td>
<td>3.3</td>
</tr>
<tr>
<td>50.6</td>
<td>0.974</td>
<td>0.167</td>
<td>0.6</td>
<td>58.12</td>
</tr>
<tr>
<td>50.1</td>
<td>0.000</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>49.6</td>
<td>-1.991</td>
<td>0.167</td>
<td>1.2</td>
<td>27.80</td>
</tr>
<tr>
<td>49.1</td>
<td>3.793</td>
<td>0.167</td>
<td>2.3</td>
<td>6.35</td>
</tr>
<tr>
<td>48.6</td>
<td>0.367</td>
<td>0.167</td>
<td>0.2</td>
<td>83.49</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Reset</th>
<th>Scale factor $[\times 10^{-4}]$</th>
<th>M.S.E. $[\times 10^{-4}]$</th>
<th>T-value</th>
<th>P-value $[^\circ]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2. corrected for periodic errors $\Delta g = 38.2015 \pm 0.0035$ $MO = 0.0032$</td>
<td>51.1</td>
<td>7.295</td>
<td>0.164</td>
<td>4.4</td>
</tr>
<tr>
<td>50.6</td>
<td>3.471</td>
<td>0.164</td>
<td>2.1</td>
<td>7.89</td>
</tr>
<tr>
<td>50.1</td>
<td>0.0</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>79.6</td>
<td>-3.124</td>
<td>0.164</td>
<td>1.9</td>
<td>10.53</td>
</tr>
<tr>
<td>49.1</td>
<td>5.352</td>
<td>0.164</td>
<td>3.3</td>
<td>1.71</td>
</tr>
<tr>
<td>48.6</td>
<td>2.222</td>
<td>0.166</td>
<td>1.3</td>
<td>22.84</td>
</tr>
</tbody>
</table>

Tab. 6.2.

Dimension of gravity values [mgal]
7. Conclusion

The measurements showed that also the D-model gravimeters of LCR are affected by periodical errors. They originate from the turns of the measuring screw and have an amplitude of 2.5 and 3.0 μgal in the D-38. In forthcoming investigations the constancy of amplitude and phase-lag in different parts of the screw will be checked.

No final decision about the gear-box generated periods of the G-258 could be made. There seems to be an amplitude of 2.7 μgal for the 1 mgal period. However the computations showed that an adjustment may result in reasonable values for amplitude and phase-lag which have to do very little with the 'true' periodical errors.

There is still to little material to make a decision about the sources and the systematics of the scale factor variations in the D-meter. There is a strong indication that they are caused by the varying reduction ratio of the lever system. Nevertheless, influences of the reset position can not yet be excluded. The magnitude of scale changes was found to be $7 \times 10^{-4}$ in the D-38. In further investigations care should be taken that the calibration ranges are not effected by the screw errors.
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Appendix

Hypothesis testing in the general linear model

Consider the model

\[ y = Ax + e, \quad y \text{ obs, } A \text{ known, } x \text{ unknown, } e \text{ errors} \]

\[ (nx1) = (nxm)(mx1) + (nx1) \]

where

\[ (2) \quad E(e) = 0, \quad D(e) = \sigma^2 G, \quad G \text{ known, rank } (G) = n, \quad \sigma^2 \text{ unknown} \]

\[ \text{rank}(A) = q < m \]

The minimisation problem

\[ (3) \quad (y - Ax)^T G^{-1} (y - Ax) = \min_x \]

is solved by

\[ (4) \quad \hat{x} = (A^T G^{-1} A)^{-1} A^T G^{-1} y \]

for any choice of the g-inverse \((A^T G^{-1} A)^{-1}\)

The residual vector \(y - A\hat{x}\) and the minimum value

\[ (5) \quad R_o^2 = (y - A\hat{x})^T G^{-1} (y - A\hat{x}) \]

\[ (6) \quad E(R_o^2) = (n - q) \sigma^2 \]

Suppose that within the model (1) we want to test the hypothesis that \(k < n\) given, lineally independent, estimable linear functions of the parameters \(x\) have some preassigned values

\[ (7) \quad H_1^T x = c_1, \quad H_2^T x = c_2, \quad \ldots, \quad H_k^T x = c_k \]

\([H_i \text{ are column vectors, } c_i \text{ scalars}]\)

Remark: Estimability of a parametric function \(H_i^T x\) can be defined in three equivalent ways

a) There exists a vector \(L\) such that \(E(L^T y) = H_i^T x\)

b) \(H_i\) can be expressed as a linear combination of the rows of \(A\)

c) \(H_i^T \hat{x}\) is independent of the choice of \((A^T G^{-1} A)^{-1}\) in (4)
Denote for shortness
\[ H = (H_1 | \ldots | H_K), \quad c = (c_1, \ldots, c_K)^T \]

Then (7) can be written
\[ (7') \quad H^T x = c \]

Let \( R^T \) be the minimum of the constrained problem corresponding to (3):
\[ \begin{align*}
(8) \quad R^T &= \min_x (y-Ax)^T G^{-1}(y-Ax) \\
&\quad \text{subject to } H^T x = c
\end{align*} \]

If the hypothesis (7) is not true, we would expect a large increase over \( R^2_0 \). In fact, it can be shown that when (7) is valid,
\[ (9) \quad \mathbb{E}(R^T - R^2_0) = K\sigma^2 \]

and when it is not
\[ (10) \quad \mathbb{E}(R^T - R^2_0) > K\sigma^2 \]

Since (6) holds independently of (7), we construct the quotient
\[ (11) \quad V = \frac{(R^2 - R^2_0)/K}{R^2_0/(n-q)} \]

and consider large values of \( V \) as evidence against the hypothesis (7).

If the error vector \( e \) is normally distributed, then
\[ (12) \quad R^2_0 \sim \sigma^2 \chi^2_{n-q} \]

where \( \chi^2_{n-q} \) denotes a stochastic variable with the chi-square distribution on \( n-q \) degrees of freedom. Further \( R^2_0 \) and \( R^T - R^2_0 \) are stochastically independent and under the hypothesis (7)
\[ (13) \quad R^T - R^2_0 \sim \sigma^2 \chi^2_K \]

Then \( V \) in (11) has the Fisher F-distribution with \( K \) and \( n-q \) degrees of freedom in the numerator and in the denominator, respectively. Tests for the hypothesis (7) can thus be constructed from tables of Fisher's F.
In this case of normally distributed errors, the test which rejects when $F$ in (11) is large, is also the likelihood ratio test for (7). It is not necessary to solve the minimum problem (8) in order to obtain $R^2_i-R^2_o$.

Denote

\[ z = H^T x, \quad \xi = z - c, \quad D = \frac{1}{\sigma^2} D(z) = H^T (A^T G^{-1} A)^{-1} H \quad \text{that is,} \]

\[ z_i (i=1, \ldots, K) \text{ are the estimates of the estimable parametric functions} \]

\[ H_i^T x, \quad \xi_i \text{ are the deviations of the estimates from the assumed values of the functions and} \]

\[ \sigma^2 D \text{ is the variance-covariance matrix of the estimates.} \]

Now it can be shown that

\[ R^2_i - R^2_o = \xi^T D^{-1} \xi \]

This, by the way, provides us with a formula for $E(R^2_i-R^2_o)$

\[ E(R^2_i-R^2_o) = K\sigma^2 + (H^T x - c)^T D^{-1} (H^T x - c) \]

where $x$ is the true (unknown) parameter vector.

In our original notation (15) reads

\[ R^2_i - R^2_o = (H^T x - c)^T (H^T (A^T G^{-1} A)^{-1} H)^{-1} (H^T x - c) \]

Consider then the special case that the $j$th component $x_j$ of the parameter vector $x$ is estimable and we want to test the hypothesis that $x_j = 0$. Then $K = 1$, $C_1 = 0$, $H_1 = (0, \ldots, 0, 1, 0, \ldots 0)$

and (17) reduces to

\[ R^2_i - R^2_o = \frac{\hat{\xi}^2_j}{(A^T G^{-1} A)^{-1}}_{jj} \]

and (11) to

\[ V = \frac{\hat{R}^2_j / (A^T G^{-1} A)^{-1}}{R^2_o / (n-q)} \]

A closer inspection shows that the test variable $V$ in (19) is the squared ratio of $\hat{\xi}_j$ and its standard error

\[ V = \frac{\hat{\xi}^2_j}{m(\hat{\xi}_j)} \]
In the case of the normal distribution it can be shown that under the null hypothesis \((x_j = 0)\)

\[
T = \frac{\hat{R}_j}{\hat{m}(R_j)} \sim t_{n-q}
\]

where \(t_{n-q}\) denotes the student t-distribution with \((n-q)\) degrees of freedom. Inference on \(x_j\) can thus be based on the t-distribution.

The computation of the test variable (11) is often presented in the form of an analysis of variance table.

<table>
<thead>
<tr>
<th>Degrees of freedom</th>
<th>Sum of squares</th>
<th>Mean square</th>
</tr>
</thead>
<tbody>
<tr>
<td>Deviation from the hypothesis</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(H^T x = c)</td>
<td>(K)</td>
<td>(R^2_1 - R^2_0)</td>
</tr>
<tr>
<td>Residual</td>
<td>(n-q)</td>
<td>(R^2_0)</td>
</tr>
<tr>
<td>Total</td>
<td>(n-q+K)</td>
<td>(R^2_1)</td>
</tr>
</tbody>
</table>

The test variable \(V\) is the ratio of the "Mean Square" values.

The appendix was written by Jaakko Mäkinen of the Finish Geodetic Institute during a DAAD grant at the Technical University of Darmstadt. Mr. Mäkinen also wrote the final version of the adjustment programm.
Nonlinearities in LaCoste and Romberg model D gravimeters determined by
the "Cloudcroft Junior" method

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Abstract

Calibration nonlinearities for three LaCoste and Romberg model D gravimeters
were determined in the laboratory to a precision of two parts in $10^6$ by the
"Cloudcroft Junior" method. The resulting interval factor values were assessed
by comparisons with values determined on a twenty-storey office tower and by
inter-comparisons among instruments during regular network surveys involving
about 1400 gravity difference observations. The laboratory calibrations were
found to describe the response of our two principal gravimeters D6 and D27
adequately until D27 was damaged in the field and returned to the manufacturer
for minor repairs. Changes in interval factor values of a few parts in $10^4$
due to the physical damage to D27 were detected both in the laboratory and in
the field. On the basis of a relatively small amount of field data a third
gravimeter, D28, appears to exhibit unexpected calibration nonlinearities which
need to be studied further. A study of the effect of reset-screw adjustments on
interval factor values showed changes of a few parts in $10^4$ which could
explain the apparent changes in linear scale factors for model D gravimeters on
the European calibration line. The results suggest that the stability of
calibration curves should be continuously monitored by solving for relative
calibrations among two or more instruments on common networks and multipoint
calibration ranges.
Introduction

The presence of calibration nonlinearities in LaCoste and Romberg Model D gravimeters at the level of a few microgals has been recognized by various users over the last few years (e.g. Steinhauser, 1978; Lambert et al., 1979, Torge and Kanngieser, 1979). In the monitoring of temporal variations of gravity, the effect of unknown calibration nonlinearities can be minimized by resetting the gravimeters to the same dial reading on each network resurvey. Over the long term, however, the effect of nonlinearities cannot be entirely avoided because instruments have a finite lifetime and nonlinearities could change due to mechanical or thermal disturbances. The availability of accurate calibration over the 200 mGal range of the gravimeter is, therefore, important for flexibility of field operations (i.e. interchanging instruments) and for improving the confidence in survey results. Crucial questions for the monitoring of the gravity field are the stability of calibration under field and instrument transport conditions and the effect on calibration of small changes of the reset screw necessary to offset drift and tares between surveys.

Calibration nonlinearities were first revealed when apparent changes in gravity difference were obtained at different position of the fine-screw while measuring a fixed gravity interval (Steinhauser, 1978; Lambert et al., 1979). Significant apparent changes in calibration with reset screw position were found (Torge and Kanngieser, 1979) when LaCoste & Romberg gravimeter D14 results were compared to established gravity intervals along the European calibration line. It was not clear whether the above apparent changes in calibration were due to a reset-screw effect or to fine-screw system nonlinearities, since the two effects cannot be unambiguously separated in the field.

The "Cloudcroft Junior" method devised by LaCoste and Romberg Inc. allows fine-screw and reset-screw nonlinearities to be studied independently. In the present experiment this method is used to examine the variations in calibration associated with the fine screw for different fixed positions of the reset screw.
Determination of Interval Factors

The calibration of a gravimeter is normally defined in terms of the ratio of the change in gravity \((\Delta g)\) to the change in dial reading \((\Delta R)\) of the gravimeter. The calibration is expressed in terms of the product of an interval factor which changes as a function of dial reading and a scale factor which does not change as a function of dial reading. The interval factors express the departure of the calibration from linearity. They are determined in the "Cloudcroft Junior" method by adding a small calibrating mass to the gravimeter beam to simulate a gravity difference of approximately 20 mGal (Figure 1). The variations in apparent gravity difference over the range of the gravimeter can be inverted to produce a set of interval factor values that approximate an interval-factor curve. The multiplying scale factor is then determined from measurements over "known" gravity differences on a calibration range.

The experimental procedure involves removing the "core" of the gravimeter from its insulated box, installing various attachments and inserting the "core" into an enclosure where it is left overnight to stabilize to its operating temperature. The attachments include a "latitude" nut that is rotated to produce an apparent increase or decrease in the reference gravity and a "calibrating mass" with a pick-up bracket for loading and unloading the gravimeter beam. The "latitude" nut allows a near null to be achieved for any position of the reset screw or of the fine screw (dial) of the gravimeter. A set of gravity "ties" at a particular interval on the dial is simulated by a series of on-off movements of the calibrating mass. Normally, eight simulated ties were performed at ten positions across the range of the dial.

Calibration data for four different LaCoste and Romberg model D gravimeters have been determined since 1976, three of them by the "Cloudcroft Junior" method. The change in interval factors across the range of the dial varies from a few parts in \(10^4\) for D13 to a few parts in \(10^3\) for D28. To a first-order
approximation the interval factors are a cubic function of dial reading (eg. Figure 2, Dragert et al., 1981).

Although the periodic screw errors are stated to be of the order of $1 \mu$Gal in the model D (Harrison and LaCoste, 1978), the existence of these and other systematic effects was investigated by making a series of "Cloudcroft Junior" determinations at 0.5 mGal intervals. Some systematic variations were seen (Figure 2), indicating that further study of short-wavelength ("circular error") effects is needed. The variations do not appear to be large enough to explain the major changes in interval-factor curves but must be taken into account when comparing interval factors determined by the "Cloudcroft Junior" method.

**Reset Effect on Calibration**

Calibrations of gravimeter D27 were determined at 20 mGal intervals by the "Cloudcroft Junior" method with the reset screw adjusted to the latitude of Charlevoix, Quebec, 47.5°N (20 clockwise turns from the setting of Austin, Texas, 30°N). The calibrations were then repeated with the reset-screw adjusted to the latitude of Vancouver Island, British Columbia, 50°N (25 clockwise turns from Austin). The determinations were made as close to the same fine-screw settings as possible to try to eliminate short-wavelength effects. The results show that, although the mean levels of the two curves only differ by one part in $10^4$, the values differ in places by more than 5 $\mu$Gal or a few parts in $10^4$ (Figure 2).

Similar tests were performed on gravimeter D28 where calibration values were studied at reset-screw positions corresponding to Austin, Texas and Charlevoix, Quebec, an adjustment of 20 turns (Figure 3). As for D27 the mean levels of the two curves differ by only one part in $10^4$, whereas the calibration for certain intervals differ by as much as 10 $\mu$Gal or five parts in $10^4$. Apparently, the basic shape of the corresponding interval-factor curves is maintained but
significant changes in the amplitude of the nonlinearities can occur due to adjustments of several turns or more of the reset screw.

Since the reset screw was adjusted by an integral number of turns in the above comparisons, the effect of possible reset-screw circular error is not included in the observed changes. A series of five "Cloudcroft Junior" determinations at $20 \times 10^3$ on the reading dial, distributed over one turn of the reset screw, did not show a significant change. However, since only one reading-dial setting was tested, significant changes at other settings cannot be ruled out.

**Comparison of Laboratory and Field Results**

Adding a mass to the gravimeter beam merely simulates the effect of changing gravity; it is necessary to show that the laboratory-derived calibrations apply for real changes in gravity. The applicability of the laboratory calibrations for gravimeter D6 was demonstrated by a comparison between "Cloudcroft Junior" derived calibration values and values derived by measuring a 20 mGal gravity difference on a twenty-storey office tower (Figure 4). Intervals of 20 mGal across the range of the fine-screw dial were sampled on the tower in a manner analogous to the "Cloudcroft Junior" procedure except for compensating adjustments of the reset screw. The two data sets compared in Figure 4 were acquired at reset-screw positions that differed by up to two or three turns. Hence, reset-screw effects could be implicated in the comparison. The results, therefore, can only be used to verify the first-order shape of the laboratory-determined curve. The discrepancies at the lower end of the dial range are probably due to a short-wavelength effect.

Further verification of the accuracy of the laboratory calibration curves was achieved by comparing results from two different gravimeters on common gravity networks (Figure 5). In this analysis, constant gravity values were derived for the network stations, laboratory calibrations were assigned to
gravimeters D6 and D27, and relative calibrations were calculated with respect to the calibration of D6 over a reference time interval of September 1976 - August 1980. The relative calibrations were determined in terms of the coefficients of a quadratic expression for Δg as a function of ΔR. The time intervals for which relative calibrations were determined were delimited by the times of accidental blows or minor factory repairs. If the nonlinear laboratory calibrations were correct and unchanging for both gravimeters, the solid lines of Figure 5 would coincide with the dashed lines. The analysis suffers from the limitation that only constant, linear and quadratic terms were included. Hence, higher order distortions would not be directly detectable.

For the time interval after the reference interval there is no evidence of any significant change in the shape of the calibration curve for gravimeter D6 that could be described by quadratic or linear terms (Figure 5, panel A). A small uniform shift of one or two parts in $10^4$ is suggested by the data, however. It is noteworthy that this small change in August 1980 was attributed to an automobile accident while the instrument was being transported. The results for D27 (Figure 5, panel B) suggest that the laboratory derived interval-factor curve should be tilted by about 2 parts in $10^4$ across the range of the instrument (an amount hardly observable by the "Cloudcroft Junior" procedure). In July 1979 a significant shift of four parts in $10^4$ occurred in the multiplying scale factor but the tilt of the interval-factor curve remained the same.

Within the limitations of the above analysis the calibrations of gravimeters D6 and D27 appear to have been stable over the last five or six years except for changes in the multiplying scale factor when the instrument accidentally received rough treatment. However, certain discrepancies in field results suggested that a more detailed analysis for possible shorter-wavelength changes in calibration was required. An analysis of the changes to D27 calibration that
occurred in June 1979 was carried out by performing a recalibration by the "Cloudcroft Junior" method at the same reset-screw position as an earlier calibration. Since the recalibration was performed using a slightly different calibrating weight than in 1978, the influence of possible short-wavelength, "circular" errors (Figure 2) could be significant in the comparison. Nevertheless, the results (Figure 6) show that, although the basic shape of the calibration remains constant, some significant short-wavelength changes have occurred. The change in the response of the gravimeter between $100\times10^3$ and $140\times10^3$ on the reading dial is supported by field results in a gravity network on Vancouver Island (Dragert et al., 1981, Figure 3).

A similar determination of relative calibrations for gravimeters D13 and D28 suffers from the relatively small amount of field data available. The analysis of D13 calibrations was terminated early when a malfunction led to its replacement by the manufacturer in 1977. Replacement gravimeter D28 was calibrated by the "Cloudcroft Junior" method in 1978 and was used later in survey work simultaneously with gravimeter D6. Preliminary results for D28 suggest so far that residual calibration nonlinearities are present but further comparisons are required to confirm it.

Conclusions

The experimental results indicate that significant changes in calibration nonlinearities can occur from one reset-screw position to another in Lacoste and Romberg model D gravimeters. Such changes could explain the apparent variability in the assumed linear calibration factor found by Torge and Kaniuge (1979) on the European calibration line. Results of comparisons in the field indicate that "Cloudcroft Junior" calibrations in the laboratory adequately describe the response to actual variations in gravity of our principal survey instruments D6 and D27. The results of the recalibration of gravimeter D27 shows that permanent changes in the higher order shape of the
calibration curve can occur due to accidental blows to the instrument in the field and/or during cleaning of the internal mechanism at the factory. Further work will be required to determine whether, in the absence of accidental mechanical shocks, the exact short-wavelength character of the calibration curves is reproduced each time the reset-screw is adjusted to within one turn of its former position.

The present analysis will be improved once a determination of the "circular error" is carried out for each instrument and appropriate corrections are made to the "Cloudcroft Junior" results. Also, laboratory calibrations show that cubic terms are necessary to delineate relative calibrations among instruments from field data.

Since LaCoste and Romberg model D gravimeters are being used to detect temporal changes in special networks, it is essential at the present time that two or more instruments be used on each survey and that the stability of calibration curves be monitored by solving for relative calibrations on common networks and multipoint calibration ranges.

Acknowledgements

The assistance of Dr. Lucien LaCoste and employees of LaCoste and Romberg, Inc., Austin, Texas in the operation of the "Cloudcroft Junior" system is gratefully acknowledged. We thank R.K. McConnell and R. Buck for help with the network analysis program and H. Dragert for data collection on network surveys. R.K. McConnell and H. Dragert made valuable suggestions which improved the manuscript. Contribution from the Earth Physics Branch No. 826.

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Figure Captions

Figure 1. LaCoste and Romberg "Cloudcroft Junior" calibration system
(reproduced by permission of LaCoste and Romberg Inc.).

Figure 2. Comparison of "Cloudcroft Junior" calibrations at reset positions 60
(filled squares) and 65 (open circles). Results of closely spaced "Cloudcroft
Junior" determinations are shown in the insert with an expanded horizontal
scale. The vertical scale in the insert is the same as in the rest of the
figure. The vertical bars denote standard errors.

Figure 3. Comparison of "Cloudcroft Junior" calibrations at reset positions 40
(filled squares) and 60 (open circles). The vertical bars denote standard
errors.
Figure 4. Comparison between "Cloudcroft Junior" (filled squares) and "Tower" (open circles) calibrations. The means of the two curves were made equal by multiplying the "tower" results by a constant.

Figure 5. Residual gravimeter nonlinearities revealed by solving for relative calibrations for gravimeters D6 and D27 over different time intervals. Statistical comparisons among the defining coefficients shows that the curves are significantly different from each other at the 95% confidence level.

Figure 6. Comparison of "Cloudcroft Junior" calibrations before (March, 1978; open circles) and after (March, 1980; filled squares) the accident of July 1979. The data were determined at reset-screw position 60. The vertical bars denote standard errors.
Figure 3

Figure 4
Abstract.

The influence of precession, nutation and polar motion on microgravimetry is studied. Precession and nutation models include equatorial and ecliptic pn-models. Emphasize is on a careful set up of a geodetic reference frame materialized by a fixed (absolute, inertial) and a moving (purely rotating) frame. It is shown that the gyroscopic and the centrifugal terms lead to a gravity effect which is by far larger than 1 μgal. Finally, for a deformable body (e.g. the earth) the divergency and the curl of the gravity vector is computed including the vorticity vector.
J. D. Boulanger, B. N. Gaipov, T. E. Demianova,  
Yu. O. Kuzmin, M. K. Kurbanov  
L. A. Marakhovskaya, S. S. Chelpanov,

LOCAL GRAVITY CHANGES AT THE ASHKHABAD GEODYNAMIC TEST-AREA

Abstract

Since 1971 repeated relative measurements of gravity are carried out at the Ashkhabad geodynamic test-area with the purpose of study of the possible gravity variations since 1971. A chain of 7 observation points are located along the Gaudan profile, which crosses the Kopetdag foredeep fault zone from north to south. The basic equipment comprises the wide-range astatised GAG-2 gravimeters for relative gravity measurements and the GS-12 N 182 gravimeter.

The values of differences of gravity acceleration as related to the initial point were determined from the averaging of measurement results obtained by means of the least squares with indirect measurements. In 1976 the analysis of data showed a change in the sign of gravity variation. The points north of the fault reveal a gravity decrease, whose minimum was in 1976, and then an increase till 1980. The reverse process is recorded at the points south of the fault.

Comparison of results of repeated levelling and of gravity anomalies made in 1976 showed, that the bench-marks rose simultaneously with the increase of gravity and that after 1976 the compared parameters were greatly reduced as the result of seismo-tectonic processes (a swarm of earthquakes in Gazli, M=7.0; 7.2; 6.5). Gravity changes and marks of points are in good agreement with the dilatancy theory.
J.D. Boulanger, B.N. Gaipov, T.E. Demianova, 
Yu.O. Kuzmin, M.K. Kurbanov 
L.A. Marakhovskaya, S.S. Chelpanov,

LOCAL GRAVITY CHANGES AT THE ASHKHABAD GEODYNAMIC TEST-AREA

Since 1965 the local gravity changes at the Ashkhabad geodynamic test-area are studied by the Institute of Seismology, Tadjik Academy of Sciences, by means of repeated gravity measurements with GS-12 N 182 gravimeter. Since 1971 these measurement are carried out jointly with the Institute of Physics of the Earth, USSR Academy of Sciences. A set of 7 observation points is located along the Gaudan profile, which crosses the zone of the Kopetdag foredeep fault from north to south. More than ten geophysical methods are used in the region of the fault to study the effect of seismotectonic processes on geophysical fields, hydrological regime, heat flow, release of radon in water, etc.

The hydrogeology of the region shows deep bedding of ground waters (75-100 m) with small changes of their level, which accounts for their weak effect on the gravity field. Drilling has shown the presence of two head water-bearing horizons at the depth of more than 1000 m and the presence of mineralised water in the fault zone at the depth of 3000 m. The body of the fault itself is tilted from south to north at about 45°. Therefore, the extreme points of the Gaudan profile 12 km long are spaced at approximately equal distances as regards the fault. The maximal difference of gravity acceleration on the profile is 53 mgal.

Gravity observations on the test-area were made jointly by two types of gravimeters in 1971, 1975, 1976, 1977 and 1980. The GAG-2 is a wide-range thermostatate astatised quartz gravimeter with torsion filament for relative gravity measurements. The observations were made according to the "double loop" program, the drift of the zero point was determined by linear interpolation. Two gravimeters were used simultaneously to shorten the time of observations at a point and to reduce the effect of tides on gravity values. In order to evaluate the effect of earth tides on the measured differences of gravity, the processing of measurements in 1971 was made with and without corrections for tides.
The maximal difference between these two variants was 7-8 mcgal. Due to the smallness of the value, the correction for tides was excluded from further processing. The values of differences in gravity as regards the initial point were determined from the equation by a program specially rendered for the computer EC 10-30 in Fortran IV. Table I shows results of adjusted differences of \( \Delta g \) and corresponding errors by years jointly for GAG-2 and GS-12 gravimeters.

The Institute of Seismology, Tajik Academy of Sciences, conducts high-precision repeated levellings with the purpose of obtaining information about recent vertical crustal movements. Levelling measurements are carried out between gravimetrical points 4-6 located south of the fault. Fig. 1 graphically shows changes of gravity as related to 1976 and to point 1 and vertical crustal movements obtained from levelling observations as related to 1976 and to geodetic benchmark 1931.

Fig. 2 shows a) changes of gravity at points from 1971 to 1980 included and b) changes in bench-marks during the same period. The following can be traced on the basis of the analysis of these graphs: if the points were divided into two groups and the first group contains the points located north of the fault (on the left-hand side of the graphs), then the second group will consist of points south of the fault (on the right-hand side of the graphs); consequently, a distinct regularity of gravity changes is observed at points in the groups. The results of averaged values of differences in gravity by points for two types of gravimeters by the two groups are shown in Fig. 3, which also demonstrates the results of averaged vertical displacements. The results of analysis revealed a change in the sign of variations of gravity and vertical crustal movements. Gravity at the group of points located north of the fault (points 1,2,3,4) was reduced by about 18 mcgal and symmetrically in relation to 1971 and 1980, whereas gravity at the group of points south of the fault (points 5,6,7) increased with the maximum in 1976 and further reduction till 1980. In this group of points the general smooth pattern as compared to the points of the first group is distorted by the data of 1971.
There are two causes for that. Firstly, in 1971 at point 5 the observations were suspended. Secondly, in 1971 there were maximal discrepancies between GAG-2 and GS-12 gravimeters in the area between the points 4-6. This might have caused the dots to jump in the group of points in 1971.

The comparison of results of repeated levellings and of gravity anomalies has revealed in 1976 the rise of bench-marks with simultaneous increase of gravity and the sudden reduction of compared parameters after 1976, caused by active seismotectonic processes (earthquake swarm in Gazli with magnitude 7.0, 7.2, 6.5). Obviously, if a part of the earth's surface, including the fault zone, is subjected to tectonic deformations, then the deformations determine the changes in density characteristics of rocks composing the body of the fault.

Gravity changes are clearly seen in Fig. 4 for 1977 and 1980 in relation to 1976. The graph vividly shows the dynamics of the increase of the amplitude of variations in time with the opposite sign on both sides of the fault line, i.e. the increase of gravity at point 4 and the decrease to points 5, 6, 7.

How can we explain the revealed jumping gravity variations at the Ashkhabad geodynamic test-area? It is known, that dilatancy might be one of the causes of local gravity changes, i.e. the growth of the volume of rocks due to deformation processes. In accordance with modern concepts, this phenomenon is associated in seismoactive regions with the seismotectonic cycle. The increase of the volume, in its turn, can produce two events, which may cause gravity variations, i.e. changes in the absolute level of points of gravity determination and changes in density due to fissuring of rock masses. In 1976 an earthquake in Gazli could have caused the rising of the absolute level of points with simultaneous increase of density of rocks in the fault zone.
Table 1.

Adjusted values of gravity differences in mGal measured by GAG-2 and GS-12 gravimeters and respective rms errors.

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<th>$\Delta g_{1-3}$</th>
<th>$\Delta g_{1-4}$</th>
<th>$\Delta g_{1-5}$</th>
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Fig. 1 Comparison between gravity changes and vertical displacements referring to epoch 1976.
Fig. 2 Gravity changes versus vertical displacements.
Fig. 3a) Gravity changes averaged for all the points to the left of the fault.

3b) Gravity changes and vertical displacements averaged for all the points to the right of the fault.
Fig. 4 Gravity changes for 1977 and 1980 in relation to 1976 and vertical displacements.
TIDAL CORRECTIONS FOR ABSOLUTE GRAVIMETRY

E. Groten, Darmstadt

Abstract. Since accuracy of about ± 2 microgal is at hand reductions and corrections applied to gravimetric measurements should have accuracies of better than ± 1 microgal. Tidal reductions, if correctly applied, yield such accuracies. Consequences are discussed.

1. Introduction

Prof. J. Faller's most recent results seem to indicate accuracy of absolute gravity measurements of the order of ± 2 microgals. Consequently, all perturbing effects should be eliminated with corresponding accuracy of ± 1 microgal and better in order to avoid accumulation of systematic distortions. Non-tidal astronomical effects such as polar motion (wobble) were reviewed in (GRODEN, 1981; 1981a) from this viewpoint.

On the other hand, using up-to-date tidal developments (such as the one by Cartwright-Edden) we are able to achieve high accuracy as demanded in high-precision gravimetry. WAHR (1979) has recently given Love-numbers for oceanless models of the Earth based, e.g., on the model 1066 A by Gilbert and Dziewonski. Consequently, for all semi-diurnal and diurnal waves corresponding gravimetric factors can be determined with relatively high accuracy. Moreover, SASAO and WAHR (1981) estimated the effect of motion connected with the nearly-diurnal free wobble which, however, is found to be relatively small. GOAD (1980) has demonstrated that for M_2 the sea tide model calculated by Schwiderski yields indirect effects which are in excellent agreement with tidal gravity data. From these results we can draw the conclusion that the total tidal effect of M_2 can be modeled quite well in gravity measurements. With forthcoming results for most of the significant tidal waves (such as O_1, K_1 and even M_f) for corresponding sea tide models of Schwiderski modeling of indirect effects in gravimetric data will no longer depend on local empirical gravimetric determinations; we can rather anticipate reliable modeling of tidal effects in gravimetry.

There might be some residual effects in coastal areas due to local sea tide perturbations which are not contained in the afore-mentioned global sea tide models. Typical examples could be the areas around the North Sea, Irish Sea, Bay
of Fundy etc. Such effects can, however, be determined empirically from a set of local tidal gravity observations. Linear super-position will then solve the problem. Insofar no principal problem is expected even in such areas.

2. Empirical parameters versus global models

This paper is presented to a workshop; consequently, it is a technical paper rather than a scientific presentation although it deals with scientific details.

In the past the tidal corrections were mostly derived by applying the empirical "local gravimetric factor"

\[ \delta = \frac{\bar{a}}{a} \]  

from which the tidal correction \( \bar{a} = \delta a \) is immediately found; the theoretical tidal amplitude \( a \) of the rigid Earth depends on the reliability of the underlying tidal model. In this way the deficiencies of the theoretical model are, to some extent, reduced. For \( \delta \) we usually get numerical values such as 1.16. In order to reduce, in addition, observation errors in the empirical determination of \( \delta \) we might observe for a longer period this enabling a tidal analysis yielding \( \delta \)-factors for the principal constituents \( (M_2, O_1, K_1 \text{ etc.}) \)

\[ \delta_i = \frac{a_i}{\bar{a}_i} \quad \text{(subscript i for a given constituent).} \]  

Such averages do no longer contain transient phenomena which can be better detected and thus incorporated in the correction if tides are recorded simultaneously with the gravity observation to be corrected.

In view of recent improved sea tide models, the rather slight local geological influence in the vertical tidal component and the good knowledge of global Earth parameters modern tidal corrections are usually derived from theoretical tidal models such as J. Wahr's model.

In order to make the basic points clear we shall simplify things somewhat in the following arguments:

With the introduction of the Normal Gravity Field 1980 a rather different concept was applied to geodesy, so that the fundamental question of "model gravity versus empirical parameters" is now of actual interest to high-precision gravimetry. The reason for preferring theoretical models (in comparison to empirical determinations) lies in the fact that long-period and other small amplitude waves need long-time records if they have to be well determined; moreover, we can now model direct tidal effects such that high precision is achieved and
and it would be good if we had one internally consistent set of model parameters entering nutation, precession, tide, diurnal-wobble and similar perturbation computations in modern geodesy.

When selecting such a model it will always, after some years, be found to be obsolete and there will, at any time, be specific deficiencies and discrepancies in comparison to reality. However, the principal question is: do we prefer using an internally consistent model (even if it deviates from reality) in high-precision gravimetry? The IAG, by adopting the Normal Gravity System 1980 has preferred a "tide-free" model which is basically fictitious (as will be shown below) and has little to do with reality.

MELCHIOR (1981) has revealed discrepancies between empirically determined gravity tide parameters and J. Wahr's aforementioned model. Such differences are anticipated because, e.g., his models are not based on most recent Earth models; for instance, Gilbert and Dziewonski's model is more than half a decade old. Moreover, MELCHIOR (ibidem) as well as HARRISON (private communication, 1981) pointed out a great number of details which have to be carefully considered when Wahr's results are applied to tidal correction work. Nevertheless, we have to decide in high-precision gravimetry which sort of tidal correction is optimal for our purposes. The fact that Wahr's work opens the way to such a tidal model correction approach was clearly pointed out by GROTEN (1981). What sort of up-to-date-modification of Wahr's numerical data and what kind of supplementary terms should be added? These are questions of practical concern to this workshop. I have defined above the gravimetric factor in a rather crude way; however, in doing so it can be considered as a quantity which absorbs (at least, to some extent) the deficiencies of the underlying theory. It has been proposed to use the record of a nearby simultaneously running tidal gravimeter for correcting the gravity readings obtained in the forthcoming D-meter campaign. Insofar the question posed above is of actual interest. Moreover, in absolute gravimetry a well defined system of reference should be used for tidal corrections. It is well known that a somewhat questionable application of Honkasalo's correction led, some years ago, to a confusion in the interpretation of absolute gravity data.

The application of Wahr's theory to tidal corrections implies a number of difficulties (inclusion of the "inertial correction", latitude dependent gravimetric factors, the inclusion of harmonics which do not exist in the tidal potential); the super-position of indirect effects according to Schwiderski's sea tide models introduces additional intricacies.
A partial alternative would be to use Melchior’s tidal maps as published in his book and elsewhere. However, in view of uncertainties in measurements such an alternative should only applied after very careful consideration of the various details and error sources in tidal data.

According to the opinion of the present author it would be optimal if the recently established working group "Standard Earth Tide Model" adopted a precise and realistic formula based on an up-dated Wahr-model which could then be used for local and global high-precision gravimetry. The selection of the specific flattening ("tide-free" or not) has to be seen in connection with the recent Normal Gravity System 1980 and geophysical needs. Details are explained below.

3. The permanent tide

The IAG adopted a resolution at the IUGG-General Assembly in Canberra (1979) according to which all gravimetric data should be reduced to a tide-free model of the Earth. Such a resolution was necessary in view of the confusion which arose when Honkasalo’s correction was applied to absolute gravity measurements. However, no specific set of data was prescribed by IAG which could lead to a tide-free model. MORITZ (1981) prefers a set of purely elastic parameters in order to eliminate the effect of the waves $M_0$ and $S_0$; GROten (1980) pointed out the problems inherent in such a procedure. In order to avoid the difficulties and uncertainties of it we proposed the elimination of the purely attractional part of the tidal potential of $M_0$ and $S_0$ thus leading to a reduction which leaves the Earth as it actually is. Such a procedure is formally equivalent to the use of "secular" Love numbers

$$h = k = 0 .$$

Other authors prefer secular Love numbers close to the fluid Love numbers

$$h_f = 1.95$$

$$k_f = 0.95$$

4. Geodetic and geophysical implications

From the view point of gravimetry alone it does not matter what model (elastic, fluid...) is applied as long as the reductions are consistently applied in satellite observations of the gravity field as well as in terrestrial gravimetry. The principal aim of the reduction (to a tide-free model), i.e. the determination of gravity being the gradient of a harmonic potential function in the space exterior to the Earth’s surface, is achieved in all of the aforementioned cases.
However, it should be kept in mind that the tide-free model deviates globally from the actual Earth by more than 0.1 m. Consequently, large-scale distance measurements such as VLBI-data must also be reduced to the same model in a consistent way. In case of VLBI-data the Love number 1 has to be introduced besides h. In addition, long-range satellite altimeter profiles have to be reduced. Therefore, the sea surface determined from reduced altimeter data is no longer identical with the actual sea surface and the corresponding geoid is the geoid of a, more or less, arbitrarily defined tide-free model but no longer the geoid of the actual Earth. The geophysical interpretation of the geoid, the sea surface etc. becomes, therefore, problematic.

In terms of gravity a deviation of 0.1 m, as mentioned above, corresponds to 30 microgals, if based on the free-air gradient. In general, the effect e is $10 \leq e \leq 30$ microgals where 10 microgal would correspond to the Bouguer gradient. This yields an "uncertainty" of the order of about 2 microgal/1000 km.

Since absolute gravity plays a predominant future role in large-scale geotectonics it will be necessary to reconcile reductions applied to geometric measurements with those applied to gravity data. Consequently, a consistent set of all three Love numbers (h, k and l) must be adopted if a great confusion should be avoided.

5. Conclusions

A recommendation should be submitted to IAG where the determination of a set of Love numbers for secular data reduction is asked for. As this set cannot be deduced from observations it has to be based on a geophysical model which is in agreement with present geophysical theories in zero-frequency seismology.

The aspects of oceanography must not be dismissed; ocean current studies etc., i.e. deviations of a somehow defined mean sea level from the geoid, have to be taken into account.

There is a significant number of people in geodesy who are in favor of a concept which does not lead to a change of the shape of the actual Earth. Consequently, such a model should also be considered where $h = k = 0$ in the secular case. A corresponding solution seems to be in agreement with the IAG-resolution.

The removal of the (direct) part of tidal potential (see Fig. 1) $V$ leads to a change of the geoid and of the sea surface; however, the separation of the actual surfaces from these counterparts are determined without hypothesis in case of $h = k = 0$ from $V/g$ where $g$ = gravity.
References


I. The Earth's reaction to small secular tidal stresses is not observable. It is generally assumed that creep exists but it is not known exactly above what lower limit of stress creep starts. These and other effects were qualitatively considered in GROTEN (1980).

Viscoelastic theories can only describe vaguely the actual behavior of real bodies such as the Earth. However, even in elementary theory of viscoelastic bodies permanent and instantaneous "elastic" parameters are well distinguished; see, e.g. RABOTNOW, Ju.N. and A.A. ILJUSCHIN, Methoden der viskoelastizitätstheorie, Hanser Verlag, München 1970 (transl. from Russian). In their rather elementary introduction "Debate about the Earth" H. TAKEUCHI, S. UYEDA and H. KANAMORI (revised ed. Freeman, Cooper and Co., San Francisco, 1970 p. 73) call such bodies (where stresses have to exceed a certain lower limit) "soft bodies". As the crust is "a soft body" and the mantle of viscous type the limit above which a fluid reaction is anticipated in case of permanent tidal stress can only be guessed.

There are advantages and disadvantages in the two basic concepts which are presently discussed as far as the permanent tide is concerned: In Heikkinen's concept the elastic theory is used in order to eliminate the deformation of the Earth (to an uncertain extent) together with the permanent tidal force itself. All other consequences are neglected: That is, neither the change in UT corresponding to the difference between "tide-free" model and actual Earth nor the corresponding difference in centrifugal force nor the consequences of the variation in centrifugal force (implications for Earth models) are taken into account.

The advantage of Heikkinen's concept lays in the fact that the flattening of the "tide-free" model is internally consistent with the external tidal forces whereas in Groten's concept the shape of the Earth is not in an equilibrium position because the permanent tidal force is eliminated (in order to end up with a harmonic external potential) but the Earth's response to it is kept as it actually is. Insofar, Heikkinen's solution is, to some extent, arbitrary and internally inconsistent as far as the implications of the elimination of the permanent tidal attraction besides the Earth's shape (and the corresponding deformation potential") are concerned.

It seems that there is no ideal solution of that problem.
II. The Gravimetric Factor:

The aforementioned definition of the Gravimetric factor is not identical with its conventional definition; HARRISON (The computation of Earth tides, Manuscript, 1981) defines the Gravimetric factor of degree \( n \) by:

\[
G_n = 1 + 2h_n / n - (n+1) k_n / n
\]

where \((k_n, h_n)\) are Love numbers of degree \( n \). The last mentioned formula is, of course, related to a spherical Love-model. Even though LOVE (Some problems of geodynamics, p. 58. Dover reprint, New-York, 1967) had already considered inertial corrections they are not included in the definition of \( G_n \). Such gravimetric factors related to models differ, of course, in general clearly from the purely empirical factor \( \delta \). This problem was basically discussed as early as in 1963 by TOMASCHEK and GROTIEN (Bull. Inf. Marées Terr. 33, Brussels, 1963) mainly in view of horizontal tide components. Even though, Wahr's "gravimetric factors" \( G_i \) are, in principle, identical (but related to more complex models) with the Love an gravimetric model factors (where \( i \) denotes again specific tidal constituents) special care is necessary in view of the various tiny differences (inclusion of inertial corrections etc.).

The rather sloppy conventional definition of the Gravimetric factor applied to eq. (1) is certainly not appropriate for high-precision work. As gravimetric drift is seldom clearly separated from long-period and similar effects we end up with a mixture of observational with modeling (spherical approximation in many cases, neglect of indirect effects etc.) deficiencies. This will, to some extent, offset the aforementioned advantages of \( \delta \) as defined above.

Dissipative forces are usually neglected in both (modeling and empirical) approaches. The maps, in (MELCHIOR, P., Tides of the planet Earth, Pergamon Press, 1978) and GROTIEN, E., Geodesy and the Earth Gravity Field, Vol. II, 1979) as well as GOAD's (1980) application of Schwiderski's sea tide model reveal that tidal phase lags (at least for \( M_2 \)) are relatively small as far indirect sea tide effects on one side and internal friction (if any) on the other side are concerned. However, in stations (such as Paris) not too far apart from the coast the tidal phase lag due to sea tides should be taken into account.

A somewhat updated formulation of the best available sea tide model (Schwiderski's model is not accepted everywhere) should be applied to GOAD's (1980) "Green's function" approach which is a modification of \( W \).
FARRELL's classical approach of 1972; J. WAHR's model should, moreover, be inspected in view of most recent results including the "tide-free" model approach (it affects the flattening etc.). From such a combination we would obtain a standard tidal map being consistent with other astronomical models. Only in exceptional cases empirical tidal formulas should be applied in the future. In a global gravimetric factor corresponding to a Wahr-type-model slight regional dependences of Love numbers \((k, h)\) on geographic latitude and longitude are omitted (longitude dependent variations are totally skipped). Those variations should be incorporated in a final standard model as was proposed by Groten (Comm. Obs. Royal Belg. A9, 228-233, 1970) in 1969. However, the present knowledge on the distribution of anomalies in the Earth's interior is not yet sufficient for such an approach. Moreover, theoretical difficulties still exist in building up a corresponding sophisticated Earth model.

III. "Inertial Correction":

This term is used in various ways. Love (1911) uses it in view of gyroscopic rigidity due to rotation; he deals with it in relation to horizontal components. Parisky denotes by this term the small acceleration of the gravity tidal station due to tidal deformation (for details see P. Melchior, Tides of the planet Earth, Pergamon Press, 1978, p. 278); in (Melchior, 1981) the notation is used in a rather general way. In my aforementioned textbook it has been used in Parisky's sense.

IV. As far as polar motion is concerned, the EIH-system should be used because this system is connected to the time and longitude systems contrary to the CIO-system.
THE "TIDE-FREE" MODEL CONCEPT; Fig. 1

- surface of actual Earth
- tide-free model surface
- level surface of tide-free model (tide-free MSL)⁺
- level surface of actual Earth (V removed)''
- level surface of W + V for rigid Earth
- level surface of W + V⁺ for actual Earth (actual MSL)⁺

Geocenter

 homogeneous ring (solar + lunar mass)
 W = Earth's gravity potential⁺
 V = tidal potential of M_o and S_o (non-harmonic)
 MSL = Mean sea level = geoid at sea

symbolic notation
V⁺ = V(1 + k - h) non-harmonic⁺
attr. ↑ def. (direct part)
V(k-h) harmonic''
Precision of the Superconducting Gravimeters Determined
by Simultaneous Measurements with Two Instruments

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ABSTRACT

Simultaneous measurements with 2, and sometimes 3, superconducting
gravimeters have been conducted at La Jolla during the past year. These
measurements revealed several sources of artificial signals such as rf
interference in the electronics, pressure effects inside of the liquid
helium vessel, and pressure induced tilts. The best signals which have
been obtained thus far show agreement between two instruments within
0.1 µgal. On a single instrument, gravity fluctuations (after removal
of tides) at periods longer than a few days are of order ±5 µgal at all of
the locations where instruments have been operated. Data from side-by-side
testing is presented.

I. PRINCIPLES OF THE SUPERCONDUCTING GRAVIMETER

The basic principle of the instrument is to suspend a superconducting
sphere over a persistent current magnet.1 The coils must be designed so as to
provide the necessary levitation force, a sufficiently small force gradient to
provide the required sensitivity, and sufficiently large horizontal restoring
forces to prevent horizontal accelerations from coupling to the vertical dis-
placement. All of these requirements are met with two coils approximately in
the Helmholtz configuration along a vertical axis with the center of the sphere
just above the plane of the upper coil. The current in each coil is adjusted
independently so that the force gradient can be adjusted to an arbitrarily small
value while the force is held at the value required to levitate the sphere.

The position of the levitated sphere is detected by a capacitance bridge. Two end plates on the top and bottom of the sphere and a ring plate around its equator form the bridge. The inner surface of these plates are contoured to form a sphere which will be concentric with the levitated sphere when the bridge is balanced. The device is operated with either electrostatic or magnetic feedback so that once the sphere is levitated and the persistent switches in the coils are closed, the variations in gravity are cancelled by the feedback force.

Since the "spring" of this device is the magnetic levitation, it is essential that variable stray fields be shielded. This is accomplished with a superconducting magnetic shield and with high permeability ferromagnetic shielding to reduce the field on the superconducting shield at the time it becomes superconducting. The gravimeter with its superconducting shield is thermally isolated inside of a vacuum container so that its temperature can be regulated electronically. This is necessary because the magnetic field of the persistent current magnet depends on temperature.

II. RESULTS OF MEASUREMENTS WITH THE INSTRUMENTS

Results obtained during the past decade have demonstrated that the instrument does provide the expected low noise and drift. Tidal records of previously unobtainable quality were used to examine ocean loading,\textsuperscript{2} to search for evidence of a universal preferred reference frame,\textsuperscript{3} and to examine the nearly diurnal free wobble.\textsuperscript{4,5} In addition a detailed study of the influence of atmospheric pressure variations on gravity was made.\textsuperscript{6,7}

For the purposes of this study group, the tides and the atmospheric pressure effect are important only because they must be accurately subtracted from the data in order to reveal the secular variations of gravity which are of interest. In Figure 2 we show this subtraction process for a record from Boulder, Colorado, and in Figure 3 we show it for a gravity "event" which occurred at The Geysers geothermal field in California. Both of these figures demonstrate the importance of
the pressure effect at the microgal level. It is the largest signal which remains after removal of the tides, and can often produce gravity fluctuations larger than 10 μgal. The ratio of gravity change to pressure change should always be approximately 0.3 μgal/mbar although this will depend somewhat on proximity to the ocean and local topography. However, a coefficient which differs from the value by more than about 30% is likely to be evidence of an instrumental pressure dependence.

Figures 1 and 2 also illustrate the importance of continuous measurements if environmental influences such as pressure, rainfall, and groundwater variations are to be identified and removed from the data. The strong correlation between pressure and gravity makes it possible to fit the pressure signal to the gravity residual and to subtract almost all of the effect from the gravity residual. Correlations with other measured quantities such as groundwater can also be subtracted. However, isolated measurements of gravity and these other variables would not allow this procedure unless the coefficients had been measured previously and were known to be independent of time. Strictly speaking this is not even true of the pressure effect since the magnitude of the coefficient depends on the surface area covered by the pressure variation. Thus, to subtract all of the pressure effect, one would need either an array of pressure sensors, or, a more sophisticated method for fitting the pressure to the gravity record.

In Figures 1 and 3 we show gravity residuals of long records after subtraction of tides and pressure effects from Boulder, Colorado and Lytle Creek, California. Both records show slow fluctuations of about ±5 μgal which appear on all records which we have obtained. Some of this is known to be caused by tilt. The tilt coefficient of the instrument is \(2 \times 10^{-4}\) μgal/(μradian)\(^2\) and tilts of 100 μradians are commonly observed on instruments which are equipped with tiltmeters. Tilts as large as several hundred μradians were observed at Lytle Creek after rainstorms. No corrections for tilt were made at Boulder. At Lytle Creek the instrument was
releveled at intervals and the lower (displaced) curve represents a linear interpolation between those points. It therefore represents a record corrected for tilt.

The steady drift on this record is the most interesting result we have obtained. The large magnitude of the observed change in gravity has forced us to examine the possibility that it is an artifact resulting from instrumental drift. However, in laboratory testing during the past year, we have never succeeded in generating a drift in this direction (it corresponds to the test mass rising) except in a case where there was a leak of helium gas into the vacuum space of the gravimeter. This causes an increase in the buoyant force on the test mass. Using a mass spectrometer leak detector over a period of several days, we were able to set an upper limit on any possible leak into the gravimeter which was two orders of magnitude below what would have been required to cause the observed drift. Gravity surveys within a fifty kilometer radius of Lytle Creek during this period did not show any significant variations at Lytle Creek. A survey to greater distance did show a change. If one assumes that our instrument was not drifting, one must conclude that we were observing an event which occurred over a sufficiently wide region as to be unobservable on the survey over a 50 km radius. Unfortunately we had no other instrument operating in Southern California during this time to confirm the result, and the observation was terminated in September 1980. Thus, we can only hope that the possibility of such a fascinating large scale phenomenon will be confirmed by future measurements.

III. RESULTS OF RECENT LABORATORY TESTING

The results described above raise some important questions about the instruments. Do the slow, 5 ugal fluctuations represent the low frequency noise limit of the instrument or a still undiscovered environmental influence on it, or do they represent true fluctuations in gravity? Can the instruments drift monotonically?
In order to answer such questions and to further improve the instruments, all of them were returned to the laboratory during the past year. We have been operating either two or three instruments simultaneously during this entire period with constant experimentation to identify possible causes of artificial signals. Several causes were identified and corrected, such as inadequate thermal contact to the superconducting magnetic shield, and ground loops in the electronics. Others are still under investigation, such as the effect of pressure variations inside of the liquid helium Dewar.

The level of stability which we expect to achieve with these instruments is illustrated by the data of Figure 4. This is a three hour segment of data from two instruments. Superimposed on the tide signal are some irregular features which appear on both records. The lower portion of the plot shows the difference between the two signals with peak excursions of about ± 0.1 μgal.

Longer records do not yet show this level of agreement between instruments. Figure 5 shows a 14-day record of the difference between two instruments (SG5 and SG2) along with the tiltmeter signals and the barometric pressure. The variation in the difference between the two gravity signals is about 2 μgal. However, the variations do not represent white noise and are caused by at least 2 factors; tilt and pressure. The tilt on SG5 was adjusted to the optimum position at the time where discontinuities in the tilt record appear. The corresponding step in the gravity difference has been removed.

Figure 6 shows the gravity residual from SG2 alone for this time period after removing tides and fitting and subtracting the barometric pressure. The coefficient was 0.29 μgal/mbar indicating that there is little or no instrumental pressure dependence. The maximum variation in this residual is less than that of the difference between SG2 and SG5 indicating that most of that difference results from instability of SG5.

No strong correlations are evident between atmospheric pressure, tilt, and gravity difference in Figure 5 even though we have measured a small response of the gravimeter
to pressure changes inside the Dewar, and the observed tilts are of the correct magnitude to produce the observed gravimeter signal differences. The pressure probably does not correlate because the instrumental dependence on pressure can not be described by a simple admittance function. The tilt does not correlate because (1) the tilt dependence of the gravimeter is quadratic, (2) both instruments are tilting and they tilt in different directions, and (3) the tilt effect is partly masked by other effects, and (4) the tiltmeters themselves, which are located directly on the gravimeters in the cryogenic environment, are producing some spurious signals. Thus these measurements do not yet identify the causes of the differences between the signals. In order to do so at least one of the possible causes must be eliminated.

Since we know that tilt is a major contributor to the noise, the instruments should be placed on tilt stabilized platforms. Although we have in the past operated two of our instruments in this way, our platforms added noise at high frequencies. Modification of the platforms has been completed and they are now being installed on the gravimeters in the laboratory. With the tilt induced signal removed, the correlation with the next largest cause of noise will become stronger.

The above discussion is intended to indicate the nature of the studies which are in progress. I have described only a very small sample of the problems which have been studied. Results such as the ones described here have demonstrated to us that we can identify and remove causes of spurious gravity signals through the technique of examining simultaneous records of 2 or more gravimeters and the variables which might influence them. In this way we expect to achieve long term instrumental stability approaching the 0.1 μgal limit shown for the short record of Figure 4.
REFERENCES


Figure 1

Gravity record from 170 days at Boulder, Colorado. The top curve is the original data where the dominant features are the one and two cycle-per-day tides. The second curve is the same data after a theoretical tide was least-squares fit and subtracted; then sinusoids at the frequencies of the nineteen largest tidal terms were also fit and subtracted. The third curve is the local barometric pressure. A strong correlation between this curve and the "de-tided" gravity is apparent. The bottom curve is the result of least-squares fitting and subtracting the barometer record from the "de-tided" gravity.
Figure 2  A 38-day data segment from The Geysers, illustrating the extraction of a barometrically adjusted residual gravity signal from the raw gravity and barometric pressure signals.
Figure 3
Gravity residual from 647 days at Lytle Creek, California. The top curve represents all of the available residual. The bottom curve is a linear interpolation of the gravity residual between the times the instrument was known to be aligned with the vertical.
Figure 4  Raw signals from SG2 and SG5 showing signals on top of tide. Lower trace is the difference between the two plotted with 0.1 μgal per division.
Figure 5  14-day record. Traces from top to bottom represent (1) barometric pressure (with tide removed), 10 mbar per division, (2) two tiltmeter axes on gravimeter SG2, 300 \(\mu\)radian per division, (3) two tiltmeter axes on SG5, 60 \(\mu\)radians per division, (4) gravity SG5 minus SG2, 3 \(\mu\)gal per division.
Figure 6  The same 14-day period as Figure 5 showing, from top to bottom,
(1) SG2 minus theoretical tide minus additional sinusoids
(2) barometric pressure minus sinusoids at 1,2,3, and 4 cycles/day
(3) SG2 residual minus the pressure and additional sinusoids
(4) SG5 minus SG2. All gravity signals 3 μgal per division.
CURRENT OBSERVATIONS OF GRAVITY VARIATIONS AND VERTICAL CRUSTAL MOVEMENTS DURING THE RECENT RIFTING PROCESS IN NORTHERN ICELAND

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1. Introduction
Precise gravity and height measurements have been carried out in the axial rift zone of northern Iceland since 1965, in order to investigate recent vertical mass shifts at that part of the Eurasian plate boundary (Pálsson and Saemundsson 1974). The investigations continue the fundamental work of Alfred Schlesener, who in 1938 first established a number of gravity stations in that area (Schlesener 1943). As, on the average, secular gravity and height variations will not exceed the few \( \mu \text{Gal} \) \( (1 \ \mu \text{Gal} = 10^{-8} \text{ms}^{-2}) \) resp. cm-order per year, high precision techniques have to be used especially in gravimetry, and systematic effects have to be controlled carefully at both methods. The observations performed during the last 15 years in Iceland partly fall in a non-active tectonic epoch (1965-1975), and partly in an active rifting episode (since 1975). This report summarizes the main results of the instrumental and methodical developments carried out at the Institut für Theoretische Geodäsie, Universität Hannover, and the results obtained before and especially during the present rifting process.

2. Developments in precision gravimetry and leveling
Gravity measurements have been carried out with LaCoste-Romberg model G and (since 1975) model D gravity meters. Developments concentrated on (Torge and Kanngieser 1980a):

- reduction of reading errors: digital voltmeter reading,

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- protection against transportation shocks: shock suppressing transport case,

- protection against wind disturbance: wind shelter,

- determination of linear and quadratic calibration terms: forth and back measurements along the European absolute calibration line between Munich and Hammerfest reduce the corresponding calibration errors to $\pm 5 \mu$gal/100 mgal,

- determination of periodic calibration terms: measurements along the 2-, 20- and 200 mgal calibration lines of the Hannover calibration system (accuracy $\pm 1 \mu$gal) reduce periodic errors to the $\pm 1 \mu$gal level,

- control of instrumental drift and error accumulation within the gravity network: establishment of a base network with overlapping ties (accuracy $\pm 3 \mu$gal) and frequent connections to the base stations,

- gravimetric earth tide reduction: introduction of regional tidal parameters observed at a temporary tidal station in northern Iceland (Torge and Wenzel 1976),

- absolute gravity reference: annual connection of the northern Iceland gravity network (base station Akureyri) to southwest Iceland (Reykjavik) and to the continent (Hannover), stability $\pm 20 \mu$gal/15 years.

As a result, the r.m.s. error of one observed gravity difference could be reduced from $\pm 19 \mu$gal (1965) to $\pm 13 \mu$gal (1980). By increasing the number of instruments from one (1965) to four (1980), the average r.m.s. error of the adjusted gravity values of one observation epoch reduced from $\pm 19 \mu$gal (1965) to $\pm 6 \mu$Gal (1980).

Height determination has been performed by technical leveling using automatic levels. From double leveling and connections to mean sea level, the accuracy of the height differences is estimated to be better than $\pm 0.01 \sqrt{\text{distance (km)}} (\text{m})$. The height reference of the different epochs has been derived either by leveling connections to the stable station Akureyri, or by using
stations with small gravity changes as reference, converting significant gravity variations with the factor \(-0.2\) mgal/m into height variations. By this procedure, the height reference could be kept within \(\pm 0.1\) m.

3. Gravity and height variations before the rifting process

The location of the gravity and partly also height profiles established in northern Iceland is given in Fig. 1 in a generalized manner (from Kanningeser 1982). Repetition surveys before the present rifting process were carried out along the west-east main profile in \(65^\circ 40'\) latitude, stretching from Akureyri (longitude \(18^\circ\) west) to Hof (\(15^\circ\) west), with an average station distance of 1 km. This profile crosses the neovolcanic zone and covers about 50 km of the adjoining western and eastern pleistocene and tertiary basalt zones. Gravity and height have been observed in 5-year intervals, 1965, 1970/71, 1975 and 1980 (see Fig. 2). The small gravity increase, indicated in the axial rift zone between 1965 and 1970 (Schleusener and Torge 1971), was confirmed at the following observation epoch 1975-1970/71 (Torge and Drewes 1977), resulting in a gravity change of appr. \(+ 0.05\) mgal/10a with respect to the western tertiary basalt zone. The maximum horizontal gradient of gravity change (0.07 mgal/10a) occurred over appr. 15 km in the Námafjall area, south of the Krafla caldera. In today's view, this variation may be explained as some kind of precursor of the present rifting episode. The 1980-1975 comparison already contains the regional long-term effect of this process. We find that an extended region of appr. 70 km width is affected by gravity decrease and correlated uplift, with a maximum of \(-0.15\) mgal/5a and \(+ 0.7\) m/5a resp. at the activated Krafla fissure zone. The central part of this zone is characterized by short-periodic variations (see next section), which accumulate to \(+ 0.2\) mgal/5a in gravity, and \(-1.1\) m/5a in elevation, respectively.

4. Gravity and height variations at the present rifting process

The Krafla fissure swarm is one of several south-north directed fissure swarm in the axial rift zone of northern Iceland, with about 80 km length and some 5 km width (see Fig. 1). Since
December 1975, this swarm shows tectonic and volcanic activity, with the Krafla caldera as the activity center. Here, rapid subsidence lasting few hours to few days occurs periodically, interrupted by uplift intervals of some months. During the subsidence events, brittle deformation is found at different parts of the fissure swarm (Pálmason 1981). These short-periodic variations are carefully monitored by Icelandic geoscientists (Björnsson et al. 1979, Johnsen et al. 1980), whereas our Institute has controlled gravity and height variations at different parts of the fissure swarm, in one-year intervals between 1975 and 1981 (Torge and Kanngieser 1980b, Torge 1981). Altogether six transverse profiles (Hverfjall; Námafjall; Gjástikki south, central, north; Lyngás) crossing, and one profile (Gjástikki-Kelduhverfi) running along the fissure zone have been established (see Fig. 1), with the observations concentrated on the Námafjall and the central Gjástikki-profiles, located appr. 10 km south resp. 15 km north of the Krafla caldera.

The observations clearly reveal the regional wandering of the swarm activity, from the north (1975/76) to the south (Námafjall 1977/78), concentrating since 1976 on the Gjástikki area. Although the observed annual variations change their magnitude with time, they show a systematic trend, characterized by gravity decrease and correlated uplift at the swarm flanks, and gravity increase and subsidence at narrow central parts of 0.5 to 2 km width, with maximum values of ± 0.1 mgal/a and ± 0.5 m/a respectively. Fig. 3 shows, as an example, the annual variations along the Námafjall profile, with decreasing but still significant values, from 1978 to 1981. Fig. 4 shows the annual variations of the central Gjástikki profile. The central part of this profile has been destroyed by lava outflow in 1980 and more extensive in 1981. The variations are decreasing since 1980. Only the gravity of station no. 556 changed strongly because of large new basalt lava masses near the station. The gravity-height variation ratio is -0.2 mgal/m on the average, which corresponds to a Bouguer-type relationship, and with extreme values at recently activated areas.

For geophysical model calculations of the subsurface mass shifts, continuous time-space models of the gravity and height variations have been developed, using least squares collocation techniques (Kanngieser 1980).
Acknowledgements

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References


Fig. 1: Gravimetric profiles in northern Iceland
Fig. 2: West-east main profile (63°40' latitude):
Gravity and height variations between 1965 and 1980
Fig. 3: Namafjall gravity profile:
Gravity and height variations between 1975 and 1981
Fig. 4: Central Gjastikki gravity profile:
Gravity and height variations between 1965 and 1981
STUDY OF SHORT TERM GRAVITY VARIATIONS
ALONG A BASELINE IN THE RHEINGRABEN

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Technische Hochschule Darmstadt

Abstract:

For the detection of short term gravity changes a one-milligal baseline in the upper Rheingraben was observed over a period of 16 months. The measurements with two LaCoste and Romberg gravity meters were repeated every two weeks. The results indicate variations in the gravity difference with a yearly period and an amplitude of 5-10 µgal. The periodical change is superposed by short term disturbances whose magnitude can reach 20 µgal.

1. Introduction

The upper Rheingraben as part of the 'World Rift System' has been the object of intensive studies. Repeated precise levellings (Schwarz, 1978) give the chance to detect tectonic as well as non-tectonic height changes. The Institute of Physical Geodesy, Darmstadt, participated in this research by performing precise gravity observations in the first order levelling network (Groten, 1974). In 1977 the 'Rheingraben gravity monitoring net' was installed (Fig. 1.1). The 23 sites are mainly points of the levelling network so that gravity- and related height changes could be detected. A more detailed description concerning the design and the results can be found in (Weichel, 1980). The measurements showed that on one hand there may be a longperiod trend at some points of about 1-3 \( \mu \text{gal} \) per year, but on the other hand there are short period oscillations in the gravity values between the epochs (the net has been observed every spring and autumn). For the study of such short term gravity variations, one gravity difference was chosen to be measured every two weeks. One reason for choosing these particular points (marked 7005 and 7200 in Fig. 1.1) was a different trend found in the adjustment computations of the gravity monitoring net. Another reason was to be able to repeat measurements with several instruments in a short interval of time, e.g. the measurements on three days during one week showed a clear negative trend of 5 \( \mu \text{gal} \).
**Fig. 1.1**

WIESBADEN
ABSOLUTE GRAVITY SITE

8450

8421

9035

10410

10400

TREND OF GRAVITY-VARIATIONS
BETWEEN 1977 AND 1979

9575

9658

SCALE
0 1 2 3 4 5

KM

RHEINGRAVEN
GRAVITY MONITORING NET

BENCHMARK
FIRST ORDER LEVELLING NET
TRIGONOMETRIC NET
GRANITE STONE

2-3±1μgal/a
1-2±1μgal/a
<0,5±1μgal/a
2. Description of the points

The sites 7005 and 7200 are levelling benchmarks. They are made of granite and sandstone respectively, their foundation is about 1.2 m deep. The position of the sites can be seen in Fig. 2.1. 7200 lies on a sediment layer consisting of sand, gravel and clay, more than 100 m thick. It is in the central part of the Rheimgraben where the most dramatic height changes take place. As can be seen in the figure, the height has changed about 1.4 cm in the last ten years. This movement is four times as big as the change between 1938 to 1967. The reason is suggested to be the increasing extraction of ground water during that time (Fahlbusch et al., 1980). The other point, 7005, lies on the western shoulder of the Rheimgraben on the so called 'Lothringische Scholle'. No height variation has been detected in the last 40 years.

3. Measuring procedure

For all measurements the LaCoste and Romberg D38 and G258 gravity meters are used. The instruments are transported by car on mainly good roads between the two sites. The distance is about 40 km. Open wooden cases with a special padding are used to prevent shocks and vibration during transportation. In order to minimize temperature effects the instruments are left in the car one night before the day of observation. In addition our G258 is put into an outer aluminium box (Gerstenecker, 1978). Each measuring day consists of 7 observations, four on 7200 and three on 7005 and lasts about eight hours. All precautions necessary in precise gravimetry are taken, e.g. waiting five minutes after unclamping
Height changes on a levelling line in the Rheingraben

Scale of height changes

\[0\]

\[1\]

\[2 \text{ cm}\]

Fig. 2.1 (taken from Schwarz, 1978)
4. Data evaluation

For the adjustment of the daily observations we use a Serbetci-Type model. Starting values are the corrected gravimeter readings. They are corrected as follows:

- conversion to approximate milligals using the manufacturer's table plus an additional scale factor
- earth tide corrections calculated by the Cartwright-Taylor-Edden tidal development
- height correction
- air-mass correction

The observation equations read

\[ v_i = Z_i - l_i \quad (i=1,n) \]

\[ u_{i,j} = Z_i - Z_j + \delta t_i - \delta t_j + \Delta g_{i,j} \quad (i=1,n-1)(j=2,n) \]

\[ \delta t_i = a_0 + a_1(t_i - t_0) + a_2(t_i - t_0)^2 + \ldots \]

\[ v_i = \text{residual of reading } i \]

\[ u_{i,j} = \text{residual 'between the stations'} \]

\[ t_i = \text{time of reading} \]

\[ t_0 = \text{starting time} \]

\[ a_i = \text{drift coefficients} \]

\[ Z_i = \text{gravimeter reading} \]

\[ \Delta g_{i,j} = \text{gravity difference} \]
The linear system is solved for the unknown parameter $z_i$, $a_i$, and the gravity differences. The $v_i$'s are the residuals of the gravi-meter readings whereas the $u_{ij}$'s are the residuals of the irregular drift behaviour. They are weighted according to empirical tests as 2:1 and it is assumed that they are uncorrelated.

5. Results

The results are listed in table 5.1 and figure 5.1. Originally it was planned to repeat the observations every 14 days. During the 16 months which are reported here it was not always possible to keep the time schedule, and greater intervals were sometimes necessary. On 3 days, only the D-38 was used. On 30.10.80 the car broke down and only 3 observations could be taken. In all observations took place on 28 days. The mean square errors obtained from the adjustment of the D-38 measurements vary between 0.4 and 6.3 $\mu$gal. The mean value is 2.5 $\mu$gal. The G-258 values for the mean square error are slightly bigger, varying from 0.9 to 6.8 $\mu$gal and are 3.0 $\mu$gal in the mean. Some of the bigger errors, and consequently the bigger differences between the instruments, are related to small accidents happening with the gravimeters. For example prior to measurement of 7.01.81 which has the biggest mean square error the D-meter got a big shock leading to a tare of 0.7 mgal. Heating failures in the G-258 lead to similar aftereffects in June 1980.
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| Mean | 1.0039 | 2.50 | 0.9998 | 3.04 | 4.34 | 1.0020 |
| St. Dev. | 0.0082 | 1.44 | 0.0062 | 1.84 | 10.06 | 0.0056 |
| M.S.E. | 0.0075 | 0.0113 | 0.0011 |

Tab. 5.1

Dimension of gravity values: mgal
Dimension of differences: ~gal
and errors
GRAVITY DIFFERENCE 7005 - 7200

Fig. 5.1

[mgal]

Rainfall

[m]

water table changes at 7200

days
There are quite large differences between the two instruments, up to 22 μgal and they are not at all normally distributed. The mean values of C and D differ about 4.3 μgal. This difference, corresponding to a scale factor of $4 \times 10^{-3}$ is caused by systematic errors. A preliminary introduction of the circular errors found on the staircase calibration line did not lead to a reduction of the discrepancy. In fact, even bigger discrepancies appeared.

6. Drift of the Gravimeters

Fig. 6.1 shows the drift curves of the instruments. The D-38 has an overall linear trend of 1 μgal per day, superposed by large nonlinear terms. We observed static drifts of −30 up to +80 μgal. The G-258 showed an abnormal drift behaviour during 1980. It started with an 0.8 mgal tare on April 10. In the following time there were drifts of 0.8 mgal per month, which were partly affected by a failure of the heating. The mean square error of the gravity differences measured at that time is higher than those of times with regular drift behaviour. This happens in spite of the very good drift control with 3 and 4 observations on the same sites. From this it is evident that our modelling of the drift with a continuous polynomial does not fit very well to the real drift behaviour. The G-meter drift obviously has a positive correlation with temperature whereas the D-meter shows only a small negative correlation.
DRIFT

LEGEND:

x G-258
O D-38
--- pressure
--- temperature

Fig. 6.1
7. **Environmental Effects**

In order to detect tectonic or related gravity changes one has to remove the known environmental effects. These are mainly groundwater and rainfall. At site 7200 the depth of the water table is about 5.5 m below the gravimeter beam. The changes of the water table are measured at intervals of 7 days in several wells. The distance of point 7200 to the nearest well is about 500 m. Using the weighted mean of the 6 nearest wells the groundwater table at the time of gravity measurements was interpolated for the site. During the period reported here it rose about 62 cm (see Fig. 5.1). The effective porosity of the ground has been determined in connection with a drilling profile. It is about 15%, the total porosity is 28%. As the gravity at 7005 is greater, the attraction because of the higher density should result in a decreased measured gravity difference. The correction calculated for a Bouguer-plate of 100 m radius amounts to a maximum of 3.76 μgal on the last observation date (Tab. 7.1 column GW). At site 7005 there is no effect of groundwater changes. The accuracy of the correction can be estimated to be better than 20%, corresponding to a maximum absolute residual of 0.7 μgal. The second correction occurs because of the changes in the humidity of the ground in the near-surface layers. Because we did not have the means to measure directly the humidity we used the rainfall records at 4 sites near the observation points. The distance between the points is only about 24 km in a direct line. Therefore the climate, temperature, humidity of the air etc., is almost the same on both sites and we can assume that the long-period changes in humidity of the ground are about the same. Their
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<td>0.0</td>
<td>0.0</td>
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<tr>
<td>18</td>
<td>23.12.1980</td>
<td>323</td>
<td>0.7682</td>
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<td>0.0</td>
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<tr>
<td>19</td>
<td>7. 1.1981</td>
<td>336</td>
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<td>-0.07</td>
<td>1.0107</td>
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<tr>
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<td>392</td>
<td>1.0022</td>
<td>-1</td>
<td>0.0</td>
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<td>4. 2.1981</td>
<td>386</td>
<td>1.0050</td>
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<td>22</td>
<td>16. 2.1981</td>
<td>380</td>
<td>1.0445</td>
<td>2</td>
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<tr>
<td>23</td>
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<td>401</td>
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<tr>
<td>24</td>
<td>1. 4.1981</td>
<td>424</td>
<td>1.0023</td>
<td>16</td>
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<td>1.0069</td>
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<tr>
<td>25</td>
<td>13. 4.1981</td>
<td>420</td>
<td>0.7974</td>
<td>21</td>
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<td>0.0</td>
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<td>1.0004</td>
<td>12</td>
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<td>27</td>
<td>5. 5.1981</td>
<td>456</td>
<td>0.7936</td>
<td>14</td>
<td>0.0</td>
<td>0.0</td>
<td>0.7905</td>
<td>1.7</td>
</tr>
<tr>
<td>28</td>
<td>13. 5.1981</td>
<td>464</td>
<td>0.9951</td>
<td>20</td>
<td>0.0</td>
<td>0.0</td>
<td>0.9790</td>
<td>3.9</td>
</tr>
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</table>

**MEAN** 1.00195  1.00370  4.6
**ST.DEV.** 0.00558  0.00573  3.0
**M.S.E.** 0.00106  0.00105  0.0

Tab. 7.1

Dimension of gravity values : mgal
Dimension of corrections : μgal
magnitude can be 15% - 20% but the effect on both sites will be almost equal. Changes in humidity differing at both sites can be caused by different amounts of rainfall in the days before observation. Therefore we took the sum of the rainfall of 4 days preceding the date of measurement plus the amount of the day itself and computed the difference between the sum collected near 7005 and 7200 respectively. This difference minus the amount of the mean evaporation was used to calculate the density changes in the first 50 cm of the ground and the resulting changes of the gravity difference. The corrections are listed in tab. 7.1. They are 0.7 μgal at a maximum and therefore should not affect the measurements significantly. This could be seen also in the correlation coefficient between rainfall and gravity difference which is only 0.13 at both stations. The values for evaporation, humidity and related quantities are taken from hydrological studies made in the Rhein-graben area (Toussaint, 1980, Schubach, 1973).

The introduction of these two corrections led to an increased mean value of the gravity difference of 1.0037±0.0011 mgal. The standard deviation was not reduced.

8. Conclusions

Fig. 8.1 shows the corrected mean values of the gravity difference together with its mean square error. The mean value of this error is 4.6 μgal with 3 μgal standard deviation. There is no indication of a secular change in the gravity difference. Instead one can assume an almost annual variation. The difference is smaller in the second quarter of the year and has a greater value between October
CORRECTED MEAN VALUES
OF GRAVITY DIFFERENCES

Date
Fig. 8.1
and February. This variation is superposed by short term disturbances with an amplitude of more than 10 µgal. They can be seen clearly in December 1980 whereas in June the inter-instrumental discrepancies allow no final decision as to what happened here. Up till now there is no explanation for these changes in gravity. In spite of the corrections applied there may be additional influences of groundwater and ground humidity, but these can not completely explain the variations. The computation of correlation coefficients with air pressure and temperature did not reveal significant results.

Furthermore the comparison of the two instruments showed again the important effects of systematic errors. The differences can not, or at least not completely, be explained by periodic errors alone.

In precise measurements at least two instruments should always be used simultaneously.

The results given here show clearly how careful one has to be in interpreting gravity changes as long as there are unexplained variations an order of magnitude bigger than the ones which can be expected from tectonic or non-tectonic gravity and height changes.
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Establishment of a LaCoste & Romberg Gravity Network in Greenland

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Abstract

The Danish Geodetic Institute has since 1976 established c. 400 LaCoste & Romberg gravity stations in Greenland, primarily in northern Greenland in connection with the major geodetic survey recently carried out. The principal aim of the gravity surveys was to provide a regional gravity coverage in northern and north-eastern Greenland in addition to the establishment of a fundamental network of gravity reference stations and tying in older gravity networks.

The LaCoste & Romberg gravity network have been connected to IGSN-stations in Denmark, Iceland, Canada (Alert) and Norway. The adjustment of the measurements indicated systematic scale errors in the LaCoste & Romberg calibration tables and the old gravity networks of 0.5 and 3 promille respectively, assuming the IGSN-scale to be correct. Due to the relatively low priority of the gravity measurements the structure of the gravity network is often suboptimal, necessitating assumptions of constant instrument drifts during longer periods. This combined with the rough arctic environment causes the standard deviation of a single measurement to become 30 - 50 μgal, somewhat larger than precision measurements in e.g. Europe, but satisfactory especially taking the huge area and the large gravity differences into account.

In the paper the gravity survey operations and adjustment method will be outlined, and a brief account of our experiences using LaCoste & Romberg gravimeters in the Arctic will be given.

1. Introduction

The Danish Geodetic Institute initiated modern LaCoste & Romberg gravity measurements in Greenland in 1976 in connection with a surveying expedition to Washington Land in northwestern Greenland, and since then gravity measurements have been performed each summer as an integrated part of the institute's other surveying activities.
Most of the gravity measurements have been made in northern and north-eastern Greenland in connection with the so-called "North Greenland Project" (see Madsen 1979), the large scale survey of the uninhabited northern areas of Greenland, executed 1976-80, covering the area from 70°N on the west coast to 76°N on the east coast, a total icefree area of 300,000 km². The principal aim of the North Greenland Project was to provide ground control for use in aerotriangulation and subsequent topographic mapping. The geodetic survey was based on doppler positioning and to a smaller extent on classical geodetic surveying. A total of c. 150 doppler stations was established and these were supplemented by a roughly equal number of barometric height determinations. The gravity measurements were made whenever possible primarily in order to obtain a regional gravity anomaly coverage to be used in geoid computations (Forsberg and Madsen, 1981), but also with the establishment of a fundamental gravity network in mind. The distribution of the gravity stations can be seen in fig. 1, which also shows the main internal and external gravimeter ties in the area. The transportation mode was airborne exclusively, using Bell Jet Ranger helicopters or a Twin Otter STOL aircraft capable of landing on unprepared natural landing strips.

In addition to the north Greenland survey the establishment of a fundamental LaCoste & Romberg gravity network, covering the whole of Greenland, has been in progress since 1976. The principal aims of this work have been to establish a network of gravity control stations to be used as a reference for future gravity surveys, onshore as well as offshore, and to remeasure stations in the old gravity network in order to be able to transform results of pre-1976 gravity measurements to the IGSN71 gravity datum. As in northern Greenland all LaCoste & Romberg gravity measurements have until now only been made in connection with other geodetic activities in Greenland, so many ties and new stations have still to be added in order to be able to talk about a "fundamental" network. Fig. 1 shows the present (1981) status of the network and ties (measured by Geodetic Institute) to external fundamental gravity stations. The network has been tied properly to the two existing IGSN71 station groups in Greenland - Thule Air Base (TAB) and Sandre Stromfjord (SFJ) - and to IGSN71 stations in Alert, Canada (ALE), Iceland (REY) and Copenhagen (CPH) in addition to ties to the Norwegian first order network in Oslo and Longyearbyen on Svalbard (SVB). A total of nine connections between Denmark/Iceland and Greenland have been obtained at present. Three different LaCoste & Romberg gravimeters
have until now been in use. Table 1 gives a short chronological summary of the measurements.

2. Reduction and adjustment methods

As a first step in the adjustment process the raw observations are converted into "reduced" observations using the LaCoste & Romberg calibration tables and the tidal correction formulas of Longman (1959) supplemented with the Honkasalo correction to eliminate Longman's time-invariant tidal effects. As earth tide factor $\delta = 1.15$ is used. The "reduced" observations are then combined in a least squares adjustment with the station gravity values, drifts, tares and (eventually) gravimeter scale factors as unknowns using the system described in detail in Forsberg (1979b). The type of observation equation used for new stations in the adjustment is

$$E\{y_i\} = \frac{1}{s_j} (e_k + a_1 (t-t_1) + b_m)$$

which is linearized to

$$E\{\Delta y_i\} = \frac{1}{s_j} \Delta e_k + \frac{t-t_1}{s_j} \Delta a_1 + \frac{1}{s_j} \Delta b_m - \frac{y_i^0}{s_j} \Delta s_j, \quad (2)$$

$$(y_i = y_i^0 + \Delta y_i \text{ etc.})$$

Here $y_i$ is the $i$'th reduced observation, $s_j$ the gravimeter scale, $e_k$ the station gravity and $a_1$ and $b_m$ drift and tare parameters respectively. The drift and tare parameters are associated with given gravimeters for given periods of time, defined on the basis of the network structure, known changes in the handling mode of the gravimeter, and systematic trends in the adjustment residuals.

The advantage of using an observation equation of type (2) compared to the adjustment of gravity differences is that the basic (reduced) reading is used directly as the observation, whereas the forming of gravity differences results in successive observations being correlated with a correlation coefficient of $-0.5$. Furthermore the normal equations associated with (2) are of a very simple nature: when the unknowns are ordered with the $\Delta e_k$'s first, the
corresponding first part of the normal equation matrix is simply a diagonal. The main drawback of (2) is the difficulty of establishing a realistic weighting. In many cases the observations will be highly correlated, e.g., for repeated measurements in a station, and in principle a non-diagonal weight matrix should thus be used, resulting in an ordinary, more or less full normal equation matrix. If, however, the network measurements are more "regular", with roughly equal time spans between the observations in the various stations, the mutual correlations will have relatively little effect and a diagonal weight matrix (initially a unit matrix) might be justified. This will especially be the case when the measurements are made in loops, starting and ending on a base station, and we for each loop solve for the drift parameters \( a_1 \) and \( b_m \). In this case we would have identical results using either of the alternative adjustment methods.

In northern Greenland the gravity network to a large degree consist of such loops, each loop representing a day trip - typically where the helicopter leaves the base camp in the morning, a number of gravity stations is measured in the day along with the other geodetic operations and finally the helicopter returns to the base camp with the gravimeter in the evening. A simple unit weight matrix has therefore been used for the adjustment, even though other parts of the Greenlandic network is less "nice". For these parts of the network longer "drift periods" \( (t_{l+1} - t_1) \) have been necessary.

To investigate the influence of such long-period constant instrument drift assumptions, table 2 shows results of the adjustment of the reasonably regular network in Peary Land, northernmost Greenland. When more drift and tare parameters are introduced into the adjustment the number of overdeterminations decrease and thus also our ability to detect gross errors and erroneous instrument behaviour, and thus the choice of average drift period should be a trade-off between a low standard deviation and a high relative redundancy for the observations (the relative redundancy is in this case simply the number of overdeterminations divided by the number of observations). Table 2 seems to indicate that drift periods of up to some days extent might be used, in spite of the well-known dependency of the drift of LaCoste & Romberg gravimeters on the mode of handling and transportation. Examples of drift curves (i.e., the change in reduced observations as a function of time) for the base stations in northern Greenland is shown in fig. 2.
3. Results of the adjustment

The combined adjustment of all measurements, using an average drift period of some days, showed a standard deviation of

$$\sigma = 0.044 \text{ mgal}$$

with reasonably "white-noise" residuals (r.m.s. variation of the residuals from year to year was 0.027 to 0.043 mgal).

In the final adjustment the IGSN71 stations were held fixed together with the Norwegian stations in Svalbard (the single tie to Svalbard showed an agreement of 0.03 mgal fixing the IGSN stations only). The scale factors for the three gravimeters were estimated to

- G 69: 1.00049
- G 466: 1.00053
- G 495: 1.00035

representing roughly a $\frac{1}{2}$ mgal error from Copenhagen ($56^\circ N$) to Thule ($76^\circ 30' N$). The LaCoste & Romberg calibration tables are thus not in correspondence with the IGSN71-scale.

Station gravity values for a few of the most important stations are shown in table 3. As a curiosity it can be mentioned that the highest gravity value, $g = 983.183.64$ mgal, occurred at Kap Bridgman ($83^\circ 24' N$, $25^\circ 32' W$), c. 100 km ESE of the north tip of Greenland, thus probably representing the highest $g$-value for any permanent marked gravity station on the northern hemisphere.

4. Comparison with older networks

Older gravity measurements in western Greenland have relied on a primary gravity network established 1950-55 by Kejlso (1958) and Svejaaard (1959), using a Frost-gravimeter. The datum of the old network were based on ship connections between Denmark and Greenland, using the "Danish Potsdam" datum (15.0 mgal above IGSN71) as reference, while the scale was determined by measurements on the later European calibration line from Copenhagen ($56^\circ N$) to Hammerfest ($74^\circ N$). LaCoste & Romberg ties to the older networks have until now...
been made in Thule, Søndre Strømfjord, Godthåb and Southern Greenland, and a plot of the differences is shown in fig. 3. The apparent trend suggests a scale error of 1.0031 in the old network. This relatively large scale error is in part due to a scale error in the pendulum measurements used as basis for the Frost-calibration, data from Kejlsø (1958) compared to the IGSN71 values (Morelli, 1974) thus indicates a 1.0018 calibration scale error, shown as a broken line in fig. 3. The discrepancy between this scale error and the apparent scale error might be due to unlinearities in the used Frost-gravimeter or an undetected gross error in the northern part of the old net, but this can not be ruled out or verified on the basis of the present available data.

5. Some brief experiences and concluding remarks

Generally the LaCoste & Romberg gravimeters have functioned excellently under the at times rough treatment unavoidable in heliborne surveys in areas like northern Greenland. The variations in temperature occurring between transportation and measurement, the air pressure variations and the vibrating environment are probably a main source of the relatively high standard deviation of 0.044 mgal obtained in the adjustment (compared to values around 0.020 mgal obtained in southern Denmark using the same gravimeters and processing), but also the suboptimal networks structure effects the results, especially when the drift is assumed constant for longer periods. The sensibility of the drift to external factors may be seen from fig. 2, where the apparent nearly level sections of the last half of the drift curve of G 466 year 1978 represents periods of bad weather, where the instrument was not moved. For the 1979 and 1980 examples of fig. 2 the instruments were "exercised" daily, and here the bad weather does not seen to "influence" the drift.

Associated with irregularities in the drift is the tares (jumps), which generally have been small. The largest tares (of the order 1 mgal) have occurred in connection with power failures, mostly when the standard NiCd-batteries used with LaCoste & Romberg occasionally ran out of power during extraordinarily long "trips". In the low temperatures of northern Greenland the storage capacity of the NiCd batteries has been lower and the power consumption of the gravimeter higher, thus necessitating more frequent change of batteries than in more temperate climates. As power source for recharging the batteries in the base camps a small Honda generator was used.
This presentation has merely been intended as a report on the present state of modern gravimetric measurements in Greenland, the measurements are planned to continue in the coming years, and the network will thus gradually improve. The gravity anomalies, which have been the primary objective of the gravity surveying, will not be treated in this paper, preliminary results have been published in reports from the Greenland Geological Survey (Forsberg, 1979, Weng, 1980, Forsberg, 1981).

Acknowledgments

H. Andersson, K. Ekholm, F. Madsen, J. Olsen and W. Weng have been in charge of the various gravity campaigns in Greenland, the last mentioned has also participated in the construction of the processing system and the management of the data.
References


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Table 1. LaCoste & Romberg gravity measurements in Greenland by the Geodetic Institute, Copenhagen

<table>
<thead>
<tr>
<th>Year</th>
<th>Number of stations</th>
<th>Number of observations</th>
<th>Gravimeters</th>
<th>Main area(s)</th>
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<tr>
<td>1976</td>
<td>27</td>
<td>125</td>
<td>G 69</td>
<td>Washington Land</td>
</tr>
<tr>
<td>1977</td>
<td>60</td>
<td>270</td>
<td>G 466</td>
<td>Western Greenland and Inglefield Land Mestersvig area (east Greenland)</td>
</tr>
<tr>
<td>1978</td>
<td>96</td>
<td>220</td>
<td>G 466</td>
<td>Peary Land (northern Greenland)</td>
</tr>
<tr>
<td>1979</td>
<td>115</td>
<td>300</td>
<td>G 466</td>
<td>Northern Greenland (north of 81°N)</td>
</tr>
<tr>
<td>1980</td>
<td>121</td>
<td>335</td>
<td>G 466, G 495</td>
<td>North-eastern Greenland</td>
</tr>
<tr>
<td>1981</td>
<td>27</td>
<td>100</td>
<td>G 466 (G 495)</td>
<td>Godthåb, Southern Greenland</td>
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</table>

Table 2. Adjustment results using various length of drift periods (Peary Land)

<table>
<thead>
<tr>
<th>Assumed average period of constant drift</th>
<th>Single observation</th>
<th>Relative redundancy</th>
<th>Standard deviation</th>
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<td>7 weeks</td>
<td>c.71</td>
<td>0.091</td>
<td></td>
</tr>
<tr>
<td>1 week</td>
<td>c.62</td>
<td>0.070</td>
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</tr>
<tr>
<td>½ week</td>
<td>c.57</td>
<td>0.053</td>
<td></td>
</tr>
<tr>
<td>Near daily</td>
<td>c.43</td>
<td>0.053</td>
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Table 3. Gravity values for some key stations in Greenland

<table>
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<tr>
<th>GI Number</th>
<th>Name</th>
<th>g(mgal)</th>
</tr>
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<tr>
<td>1001</td>
<td>Station Nord (astronomical pillar)</td>
<td>983 068.12</td>
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<tr>
<td>3358</td>
<td>Thule Air Base (hospital)</td>
<td>982 918.16</td>
</tr>
<tr>
<td>41100</td>
<td>Mestersvig (airport)</td>
<td>982 682.88</td>
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<tr>
<td>67701</td>
<td>Søndre Stromfjord M (hotel)</td>
<td>982 368.47</td>
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<td>78104</td>
<td>Godthåb (harbour)</td>
<td>982 191.86</td>
</tr>
<tr>
<td>88109</td>
<td>Narssaq (church)</td>
<td>981 957.29</td>
</tr>
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</table>
Fig. 1. LaCoste & Romberg gravity network, main internal and external ties and all stations in northern Greenland shown. Index to placenames mentioned in the text: 1 Marsaung, 2 Godshåb, 3 Westerøg, 4 Engelsfield Land, 5 Washington Land, 6 Peary Land, 7 Station Nord, 8 Kap Bridgman.
Fig. 2. Examples of gravimeter drift curves determined at the operational base stations in northern Greenland. The apparent jumps on day 4 and 12 in 1979 are due to power failures.
Fig. 3. Differences between gravity values of the old network (Frost) and the LaCoste & Romberg network (IGSN scale) on the western coast of Greenland, as a function of the LC&R gravity values. The broken line represents the known scale error in the original calibration of the Frost gravimeter.
Practical Formulae for Free Fall

Absolute Gravity and Gravity Gradient Measurement

by

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Contribution of the Earth Physics Branch No. ___
ABSTRACT

Equations of free fall, accounting for Coriolis forces and constant vertical gravity gradient, are derived by application of the principles of Classical Mechanics (Goldstein, 1980). A least squares fitting of the observed height of the falling body to a polynomial in time is proposed. Formulae for computing gravity and gravity gradient from the polynomial coefficients are established.
INTRODUCTION

For several decades absolute gravity measurements have been envisaged and realized using free falling objects (Volet, 1947), (Sakuma, 1971), (Hammond, 1978). Our purpose is to derive practical formulae for absolute gravity reduction from free fall measurements \((t_i, z_i)\) \(i=1, \ldots, m\) where \(t_i\) are the measured times corresponding to the vertical coordinate values \(z_i\). At this stage of the theory we assume that the measurements are free from instrumental errors and that the corresponding times \(t_i\) and elevations \(z_i\) rigourously satisfy the equations of motion which are derived in the present paper. The practical formulae will be obtained from a Taylor expansion of the solution \(z(t)\) of the equations of motion. Terms beyond the fifth degree are discarded then gravity \(g\) and gravity gradient \(n\) are expressed in terms of the remaining coefficients. Least squares adjustment is proposed for the practical computation of these coefficients from observations. A similar procedure was used in the past (Hammond and Iliff, 1978) involving only terms up to degree two, neglecting Coriolis forces and gravity gradient. These authors pointed out that inclusion of gravity gradient was weakening the least squares determination of \(g\). It is already interesting to make the distinction with our procedure. In fact we are proposing to make the least squares fit of a polynomial of degree five to the observed time series \((t_i, z_i)\) \(i=1, \ldots, m\) and such a computation should be a very stable one provided appropriate methods are used. Then gravity and gravity gradient are computed from these coefficients by straightforward algebra.
THEORY

Let us apply the law of Classical Mechanics (Goldstein, 1980) to a freely moving point mass \( m \) in a reference frame \( Axyz \) fixed with respect to the earth's crust, (figure 1) under the influence of some gravitational field. Denoting by \( \Omega \) the earth instantaneous angular velocity vector and \( T \) its center of mass, absolute acceleration \( \ddot{\gamma}_M \) of point \( M \) having mass \( m \) is given by (Mandel, 1968)

\[
\ddot{\gamma}_M = \dot{\gamma}_T + \frac{d\Omega}{dt} \times \nabla M + \Omega \times (\Omega \times \nabla M) + \dot{\gamma}_{MT} + 2\Omega \times \nabla_{MT},
\]

(1)

where \( \times \) is vector cross-product, \( \dot{\gamma}_T \) the acceleration of the earth's center of mass \( T \), \( \ddot{\gamma}_{MT} \) relative acceleration of \( M \) and \( \nabla_{MT} \) relative velocity of \( M \) both in reference frame \( Axyz \). Assuming for our theoretical derivation \( \frac{d\Omega}{dt} = 0 \) and denoting by \( \dot{\phi}_M \) and \( \phi_M \) the earth and other celestial bodies gravitational fields respectively, we apply Newton's Second Law of Motion (Goldstein, 1980; p. 1) and using (1) we have

\[
\ddot{\gamma}_M + \dot{\phi}_M - \dot{\gamma}_T - \Omega \times (\Omega \times \nabla M) = \ddot{\gamma}_{MT} + 2\Omega \times \nabla_{MT}.
\]

(2)

In the present theory, we assume the left-hand side of (2) to be a constant vector \( \ddot{\gamma} \) independent of time and position plus a linear term \( z\hat{n} \) where \( \hat{n} \) is the vertical gravity gradient in \( A \) and \( z \) the elevation of point \( M \). \( \hat{n} \) is assumed to be a constant in the sequel and \( \ddot{\gamma} \) is the gravity vector at point \( A \). Equation (2) becomes:

\[
\ddot{\gamma} + z\hat{n} = \ddot{\gamma}_{MT} + 2\Omega \times \nabla_{MT}.
\]

(3)
Choosing $A_z$ parallel to $\bar{g}$ in the opposite direction and $A_x$ towards instantaneous North and finally $A_y$ to make $A_{xyz}$ a right handed cartesian reference frame, $\vec{\Omega}$ is contained in the instantaneous astronomical meridian plane $A_{xz}$, making the angle $L$ (latitude) with $A_x$ (figure 1).

**Figure 1.** $A_{xyz}$ reference frame fixed with respect to earth crust

Expressing (3) with coordinates $x$, $y$, $z$, their first and second derivatives with respect to time being denoted by dots and double dots respectively, we have:

\begin{align*}
\ddot{x} - 2\dot{y}\Omega \sin L &= 0, \\
\ddot{y} + 2\dot{x}\Omega \sin L - 2\dot{z}\Omega \cos L &= 0, \\
\ddot{z} + 2\dot{y}\Omega \cos L &= -g + nz,
\end{align*}

where $g$ is the third component of $\vec{g}$ in $A_{xyz}$ and $n = dg/dz$ its vertical gradient component.
Let us solve this system of three linear differential equations
of the second order with constant coefficients and obtain the complete
solution for the yet unknown function \( z(t) \). Integrating (4) yields an
expression of \( \dot{x} \) in terms of \( y \). That expression substituted in (5) yields
\( \dot{z} \) in terms of \( \ddot{y} \) and \( y \). Differentiating once yields \( \dot{z} \) and once more \( \ddot{z} \). Differen-
tiating (6) with respect to time and substituting \( \dot{z} \) and \( \ddot{z} \) previously
obtained in terms of \( y \), \( \ddot{y} \) and \( \dddot{y} \), we obtain the following equation:

\[
\dddot{y} + \dddot{y}(4\omega^2 - \eta) + y(-4\omega^2 \sin^2 \Lambda n) + (-2\Omega \sin \Lambda (\dot{x}_s - 2\omega \Omega \sin \Lambda n)) = 0. \tag{7}
\]

The solution of (7) is

\[
y(t) = (y_s - \dot{x}_s / 2\Omega \sin \Lambda) + C_2 e^{ft} + C_3 e^{-ft} + C_4 \cos \omega t + C_5 \sin \omega t, \tag{8}
\]

where \( f^2 \) and \( \omega^2 \) are the two real roots of the characteristic equation
of (7) and \( C_2 \), \( C_3 \), \( C_4 \), \( C_5 \), \( y_s \) and \( \dot{x}_s \) are integration constants (see
Appendix A).

We deduce

\[
z(t) = C_6 + C_2 J(f)e^{ft} - C_3 J(f)e^{-ft}
+ C_5 K(\omega) \cos \omega t - C_4 K(\omega) \sin \omega t, \tag{9}
\]

\[
x(t) = C_6
+ 2\Omega \sin \Lambda \left((C_2 e^{ft} - C_3 e^{-ft})/f - (C_5 \cos \omega t - C_4 \sin \omega t)/\omega\right), \tag{10}
\]

where \( C_6 \) and \( C_7 \) are integration constants and

\[
J(f) = (f/2\Omega \cos \Lambda) + (2\Omega \sin^2 \Lambda / f \cos \Lambda), \tag{11}
\]

\[
K(\omega) = (\omega/2\Omega \cos \Lambda) - (2\Omega \sin^2 \Lambda / \omega \cos \Lambda). \tag{12}
\]

These formulae are derived in Appendix A.
Since \( z(t) \) will be used in the sequel as our law of motion, for practical purposes we wish to have it in terms of powers of \( t \). Taylor expansion of equation (9) up to the fifth degree yields (see Appendix B for the computational details)

\[
z(t) = z_0 + \dot{z}_0 t + \left( -\frac{1}{2}g + \frac{1}{2}z_0 n - \dot{y}_0 \Omega \cos L \right) t^2
+ \left( \frac{2}{6}g^2 \cos L (\dot{x}_0 \sin L - \ddot{z}_0 \cos L) + \frac{1}{6} \dot{z}_0 n \right) t^3
+ \left( \frac{1}{6}g^3 \cos L (\dot{y}_0 + g \cos L / 2 \Omega) + \frac{n (z_0 n - g)}{24} \right) t^4
- n \Omega \cos L (\dot{x}_0 \sin L + \dot{z}_0 \cos L) / 6 t^4
+ \left( \frac{-2}{15}g^4 \cos L (\dot{x}_0 \sin L - \ddot{z}_0 \cos L) + \frac{g^2 n^2 (\sin L - 2 \ddot{z}_0 \cos L) / 30 + z_0^2 n / 120) \right) t^5,
\]

(13)

where \( z_0, \dot{z}_0, \dot{y}_0 \) are initial values at time \( t = 0 \).

This type of polynomial equation will be used as observation equation in the least squares adjustment.

Practical formulae for gravity \((g)\) and vertical gravity gradient \((n)\)

In order to use formula (13) we assume that our theory is accurate enough so that systematic errors arising from the inadequacy of theory with reality are much smaller than instrumental errors of today's existing apparatus (1981). Then for our subsequent derivations we can assume that, besides instrumental errors, observations are rigourously described by formula (13). The first step of our proposed procedure is to adjust by least squares the time series \((t_i, z_i) i=1, \ldots, m\) with the mathematical model

\[
z(t) = A + Bt + Ct^2 + Dt^3 + Et^4 + Ft^5,
\]

(14)
Step two will consist of performing spectral analysis by least squares (Vaníček, 1971) of the residuals $s_i$ given by

$$s_i = z_i - \left( A + Bt_i + Ct_i^2 + Dt_i^3 + Et_i^4 + Ft_i^5 \right),$$

(15)

where $\hat{A}, \hat{B}, \hat{C}, \hat{D}, \hat{E}$ and $\hat{F}$ are least squares estimates of parameters $A, B, C, D, E$ and $F$ of formula (14). The purpose of that spectral analysis is to test the residuals for white noise (Jeudy, 1981; ch. V) and to compute their spectral representation in case white noise hypothesis is rejected. In practice, some residual effect due to microseisms may be present and we will obtain the least squares compensated values of the residuals

$$\hat{s}_i = \hat{z}_i + \sum_{k=1}^{m} \left( \hat{a}_k \cos \hat{\omega}_k t_i + \hat{b}_k \sin \hat{\omega}_k t_i \right),$$

(16)

where $\hat{s}_i$ is white noise and where $\hat{\omega}_k, \hat{\omega}_k$ are parameters computed by a simultaneous least squares adjustment of all the $m$ frequencies pointed out by the spectral analysis.

Step three consists of computing the corrected time-series $\tilde{z}_i$ which is least squares fitted to polynomial of formula (14). We now see that an iterative procedure can be adopted where differences between the original time-series $z_i$ and the adjusted polynomial of step three are calculated and again fitted by least squares to mathematical model (16). Then computing the corrected time-series as in step three, we adjust polynomial of formula (14). Repeating that procedure until convergence will give the solution which would be obtained from simultaneous adjustment of polynomial (14) plus spectral representation (16). The advantage of such a procedure, known as cyclic descent (Bloomfield, 1976; p. 22) is to avoid the possible
instability which may be encountered in a simultaneous adjustment.

Once all this set of computations is achieved we have the best possible estimates \( a, b, c, d, e \) and \( f \) of coefficients \( A, B, C, D, E \) and \( F \) in formula (14). Before using these estimates to derive formulae for gravity \( g \) and gravity gradient \( n \), we can simplify our polynomial expression in using the translations \( \tau \) and \( \zeta \) along the axis \( t \) and \( z \) respectively such that the constant term and the coefficient of the first power of time disappears. Putting \( z = z' + \zeta \) and \( t = t' + \tau \), \( \zeta \) and \( \tau \) are computed by writing:

\[
z' + \zeta = a + b \left( t' + \tau \right) + c \left( t' + \tau \right)^2 + d \left( t' + \tau \right)^3 + e \left( t' + \tau \right)^4 + f \left( t' + \tau \right)^5 , \tag{17}
\]

so that

\[
\zeta = a + b \tau + c \tau^2 + d \tau^3 + e \tau^4 + f \tau^5 , \tag{18}
\]

where \( \tau \) is a root of equation

\[
b + 2c\tau + 3d\tau^2 + 4e\tau^3 + 5f \tau^4 = 0 . \tag{19}
\]

Equation (19) means that \( \dot{z} \), projection of point mass \( m \) velocity \( \vec{v}_z \) along \( z' \) axis is zero (see figure 2). Finally we obtain the polynomial representation

\[
z' = c' \left( t' \right)^2 + d' \left( t' \right)^3 + e' \left( t' \right)^4 + f' \left( t' \right)^5 , \tag{20}
\]
where coefficients $c'$, $d'$, $e'$ and $f'$ are functions of $c, d, e, f$ and $\tau$.

Identifying (20) with (13) after changing coordinates $x, y, z$ for $x', y', z'$ and putting $z'_0 = z''_0 = 0$ where $x'$ and $y'$ axis are defined in figure 3, we have

$$c' = -\frac{x}{2} - y'_{\omega} \cos L,$$  \hspace{1cm} (21)

$$d' = 2 \frac{n^2}{3} \dot{x}'_0 \cos L \sin L,$$ \hspace{1cm} (22)

$$e' = \frac{n^3}{3} \cos L \left( y'_{\omega} + g \frac{\cos L}{2\Omega} \right) - \frac{n}{12} \left( \frac{\dot{g}}{2} + y'_{\omega} \Omega \cos L \right),$$ \hspace{1cm} (23)

$$f' = 2 \frac{n^2}{15} \dot{x}'_0 \cos L \sin L \left( \frac{n}{4} - \dot{\omega}^2 \right).$$ \hspace{1cm} (24)
We obtain a system of four equations (21), (22), (23) and (24) with four unknowns $g$, $\dot{y}$, $\dot{x}$, and $n$. In first approximation since $\dot{x}$ and $\dot{y}$ will be small non-zero initial velocities due to some instrumental perturbations, we have the solution $g=-2c'$ as obtained from (21) and $n=12e'/c'$ as obtained from (23) using the approximate value of $g$ here above. For the rigorous solution we see a problem when $L=0$, then $d'=f'=0$ and $g$ cannot be computed separately but only $g+2\dot{y}\Omega$.

Unless $\dot{y}$ is derived from other observations (such as measurements of east-west deflection), $g$ cannot be evaluated rigorously since in practice $\dot{y}$ will never be rigorously equal to zero. In this case, we obtain from (23) the gravity gradient rigorously $n=(12e'/c')+4\Omega^2$.

In the general case ($L\neq0$), the rigorous solution for $g$ and $n$ is the following:

\[
\begin{align*}
    n &= 20f' + 4\Omega^2 \\
    d' \\
    g &= -2c' - 2\left(\frac{c'\Omega^2 \cos^2 L + 6e' - nc'/2}{\Omega^2 \sin^2 L}\right) \\
\end{align*}
\]

If $\dot{y}$ can be derived from other observations, then better formulae will be

\[
\begin{align*}
    g &= -2c' - 2\dot{y}\Omega \cos L \\
    n &= \frac{12e'}{c'} + 4\Omega^2 \cos L \left(\cos L - \dot{y}\Omega \sin L \right) \\
\end{align*}
\]

In practice (28) can be approximated by

\[
\begin{align*}
    n &= \frac{12e'}{c'} + 4\Omega^2 \cos^2 L \\
\end{align*}
\]
Figure 3. Coordinate changes for x, y and z.

Legend. (Γ) is trajectory of point mass (m) in space. It is not a straight vertical because of deviation x and y (see formulae (8) and (10)). For the purpose of illustration we have exaggerated (Γ) curvature. At time $t'=0 (t=T<0)$, point mass (m) is in O' with velocity $\vec{V}_0$ contained in horizontal plane of coordinates O'x'y' ($\dot{z}'=0$). At time $t=0 (t'=-T>0)$ point mass (m) is in O with velocity $\vec{V}_0$. 

Similarly we could define $\xi$ and $\eta$ by $x=\xi+x'$ and $y=\eta+y'$ where $\xi=x(\tau)$ and $\eta=y(\tau)$.

**Precision estimation**

First, polynomial coefficients significance has to be tested (Hamilton, 1964; p. 169). Using the variance-covariance matrix of the least squares adjustment, one can easily deduce the variance of $g$ and $n$ given by (25) and (26) respectively (Mikhail, 1976; p. 161).

**Conclusion**

According to the principle of Classical Mechanics (Goldstein, 1980), the fundamental free fall formula (13) was derived. A procedure for computing the best polynomial representation of a set of free fall measurements was proposed and the basic equations for gravity (25) and gravity gradient (26) were derived. Also a procedure for precision estimation was proposed. It appears in the discussion of the obtained formulae that some implementation in the existing free fall gravimeters could be made to measure east-west deflection during free fall and deduce a good value for initial velocity east-west component at time zero.

**Acknowledgements**

Useful comments by Drs. R. Simard and A. Lambert are kindly acknowledged.
REFERENCES


APPENDIX A

A 1. Solution of equation (7)

Characteristic equation is the following:

\[ r^4 + r^2 \left( 4\omega^2 - \omega \right) + \left( -4\omega^2 \sin^2 L \right) = 0 \]

Putting \( X = r^2 \) we obtain two real roots for \( X \) denoted by \( \omega^2 \) and \( -\omega^2 \) and this yields formula (8).

A 2. Solution for \( x(t) \) and \( z(t) \)

From (4) we have

\[ \dot{x}(t) = 2\omega \sin L \left( (y_x - \dot{x}_x/2\omega \sin L) + C_2 e^{ft} + C_3 e^{-ft} + C_4 \cos \omega t + C_5 \sin \omega t \right) \]

\[ + \dot{x}_x - 2y_x \omega \sin L. \]

Then follows formula (10). For computing \( z(t) \), from (5) we have

\[ z(t) = \frac{1}{2\omega \cos L} \left( 2\omega \sin L (C_7 + 2\omega \sin L ((C_2 e^{ft} - C_3 e^{-ft})/f \right) \]

\[ - (C_2 \cos \omega t - C_4 \sin \omega t)/\omega) + C_7 e^{ft} - C_3 e^{-ft} \]

\[ - C_4 \omega \sin \omega t + C_5 \omega \cos \omega t + C_6'. \]

Putting \( t \omega L = C_7 + C_6' = C_6 \) yields formula (9) where \( J(f) \) and \( K(\omega) \) are easy to derive.
APPENDIX B

B.1. Equations for integration constants

In order to derive the coefficients of the Taylor expansion of \( z(t) \), we need the equations defining the integration constants. We have

\[
x_0 = C_7 + 2\Omega \sin L \left( \frac{C_2}{\bar{f}} - \frac{C_3}{f^2} - \frac{C_5}{\omega} \right), \tag{21}
\]

\[
\dot{x}_0 = 2\Omega \sin L \left( C_2 + C_3 + C_4 \right), \tag{22}
\]

\[
\ddot{x}_0 = C_2 \bar{f} - C_3 \bar{f}^2 + C_5 \omega, \tag{23}
\]

\[
z_0 = C_6 + C_2 \bar{J}(\bar{f}) - C_3 \bar{J}(\bar{f}) + C_5 K(\omega), \tag{24}
\]

\[
\dot{z}_0 = C_2 \bar{J}(\bar{f}) + C_3 \bar{J}(\bar{f}) - C_4 \omega K(\omega), \tag{25}
\]

and from (6), \( C_6 = \frac{R}{n} \)

B.2. Taylor expansion of \( z(t) \)

1) Constant term

From (9), we deduce that the constant term is

\[
C_6 + (C_2 - C_3) \bar{J}(\bar{f}) + C_5 K(\omega) = z, \quad \text{from (24).}
\]

2) Coefficient of \( t \)

From (9), it is

\[
C_2 \bar{J}(\bar{f}) + C_3 \bar{J}(\bar{f}) - C_4 \omega K(\omega) = \dot{z}, \quad \text{from (25).}
\]

3) Coefficient of \( t^2 \)

From (9), it is

\[
\frac{1}{2} \left( C_2 \bar{J}(\bar{f})^2 - C_3 \bar{J}(\bar{f})^2 - C_5 K(\omega) \omega^2 \right) =
\]
\[ \frac{1}{2} \left[ (C_2 - C_3) f^2 J(f) - C_5 \omega^2 K(\omega) \right]. \] From (23),

\[ (C_2 - C_3) = (\dot{y}_s - \omega C_z)/f \]
and substituting in the previous formula, coefficient of \( t^2 \) becomes

\[ \frac{1}{2} \left( (\dot{y}_s - \omega C_z) f J(f) - C_5 \omega^2 K(\omega) \right) = \]

\[ \frac{1}{2} \left( \dot{y}_s f J(f) - \omega C_5 (f J(f) + \omega K(\omega)) \right). \] (26)

Let us evaluate separately \( f J(f) + \omega K(\omega) \), we have from (11) and (12)

\[ f J(f) + \omega K(\omega) = \frac{f^2 + \omega^2}{2\Omega \cos L}. \] (27)

From (23) and (24), we have

\[ C_5 = \frac{1}{(\omega J(f) - f z_s + \omega g/n) / (\omega J(f) - f K(\omega))}. \] (28)

But \( \omega J(f) - f K(\omega) = 2\Omega \sin^2 L \frac{(\omega^2 + f^2)/f \omega \cos L}{f \omega \cos L} \). (29)

Using (27), (28) and (29), (26) becomes

\[ \frac{1}{2} \left[ (\dot{y}_s \omega f J(f) - \dot{y}_s f^2 J(f) K(\omega) - \omega f \dot{y}_s J^2(f) - \omega^2 \dot{y}_s J(f) K(\omega)) \right. \]

\[ \left. (\omega J(f) - f K(\omega))^{-1} + f^2 \omega^2 z_s / 4\Omega^2 \sin^2 L \right. \]

\[ - f^2 g/4\Omega^2 \sin^2 L \right]. \] (30)

From §41 we have

\[ f^2 - \omega^2 = -4\Omega^2 + n, \] (31)

\[ -f^2 \omega^2 = -4\Omega^2 \sin^2 L. \] (32)

From (11) and (12) we have

\[ J(f)K(\omega) = \frac{\omega f}{4\Omega^2 \cos^2 L} - \frac{4\Omega^2 \sin^4 L}{\omega f \cos^2 L} - \frac{(f^2 - \omega^2) \sin^2 L}{\omega f \cos^2 L} \]

\[ = (\omega^2 f^2 - 16n^4 \sin^4 L - 4\Omega^2 (-4\Omega^2 + n) \sin^2 L) \]

\[ (4\Omega^2 \omega f \cos^2 L)^{-1} \] (33)

\[ = 4\Omega^2 \sin^2 L/\omega f. \] (34)
Using (34), (29) and (32), (30) reduces to
\[-\frac{1}{2} g + \frac{1}{2} \dot{x}_0 \omega^2 - \dot{\omega} \Omega \cos L.\]

iv) **Coefficient of \( t^3 \)** From (9) it is
\[
\frac{1}{6} \left( C_2 f^3 J(f) + C_3 f^3 J(f) + C_4 \omega^4 K(\omega) \right) =
\]
\[
(1/6) \left( f^3 J(f) \left( \dot{x}_0 / 2\Omega \sin L - C_4 \right) + C_4 \omega^3 K(\omega) \right),
\]
(35)
after using (22). From (22) and (25) we have
\[
C_4 = \left( -\ddot{x}_0 + \dot{x}_0 fJ(f) / 2\Omega \sin L \right) / (fJ(f) + \omega K(\omega)).
\]
(36)
Using (36), (35) becomes
\[
(\dot{x}_0 / 12 \Omega \sin L) \left( f^3 J(f) - fJ(f) \left( f^3 J(f) - \omega^3 K(\omega) \right) / (fJ(f) + \omega K(\omega)) \right) + \ddot{z}_0 (f^3 J(f) - \omega^3 K(\omega)) / 6 (fJ(f) + \omega K(\omega)).
\]
(37)
We compute
\[
f^3 J(f) - \omega^3 K(\omega) = \frac{f^4 - \omega^4}{2 \Omega \cos L} + \frac{(f^2 + \omega^2)}{2 \Omega \cos L} \frac{2 \Omega \sin^2 L}{\cos L}
\]
\[
= \frac{(f^2 + \omega^2)}{2 \Omega \cos L} \left( -4 \Omega^2 + n \right) + 4 \Omega^2 \sin^2 L
\]
\[
= (f^2 + \omega^2) \left( n - 4 \Omega^2 \cos^2 L \right) / 2 \Omega \cos L.
\]
(38)
Using (34), (27) and (38), (37) becomes
\[
(2/3) \Omega^2 \cos L (\dot{x}_0 \sin L - \ddot{z}_0 \cos L) + \ddot{z}_0 n / 6
\]
(39)

v) **Coefficient of \( t^4 \)** From (9), it is
\[
(C_2 f^4 J(f) - C_3 f^4 J(f) + C_4 \omega^4 K(\omega) \right) / 24,
\]
(40)
using (23), (40) becomes
\[
(f^3 J(f) (\dot{y}_0 - \omega C_3) + C_4 \omega^4 K(\omega)) / 24,
\]
(41)
Using (28), (41) becomes

\[ e^3 J(\xi) \frac{\dot{y}_z}{24} + \frac{\omega (\omega^2 K(\omega) - f^3 J(\xi))}{24 (\omega J(\xi) - \xi K(\omega))} \left( \dot{x}_z J(\xi) - f_z + \frac{fg}{n} \right). \]  

(42)

Using (38) and (29), (42) becomes

\[ \ddot{y}_z \left( \frac{\omega^4 - f^4}{24} \right) J(\xi) K(\omega) \frac{f_\omega \cos L}{2 \Omega \sin^2 L} \right. 

\left. + \frac{e^2 \omega^2}{24} \frac{n-4\Omega^2 \cos^2 L}{4\Omega^2 \sin^2 L} \right) 

\left. + \frac{e^2 \omega^2}{24} \frac{n-4\Omega^2 \cos^2 L}{24n} \right) \frac{e^2 \omega^2 (n-4\Omega^2 \cos^2 L)}{4\Omega^2 \sin^2 L}. \]  

(43)

Using (32), (31) and (34), (43) becomes

\[ \ddot{y}_z \left( 4\Omega^2 - n \right) \Omega \cos L / 12 + (n-4\Omega^2 \cos^2 \omega) (\dot{z}_z + \frac{g}{24}). \]  

(44)

(44) is equal to the coefficient of \( t^4 \) in (13).

v) Coefficient of \( t^5 \). From (9), it is

\[ (C_2 J(\xi) f^5 + C_3 J(\xi) f^5 - C_4 K(\omega) n^5) / 120. \]  

(45)

Using (22), (45) becomes

\[ (\dot{x}_z / 2 \Omega \sin L - C_4) J(\xi) f^5 - C_4 K(\omega) n^5) / 120. \]  

(46)

Using (36), (46) becomes

\[ \dot{x}_z f J(\xi) K(\omega) \left( f^4 - \omega^4 / 24 \Omega \sin L \right) \left( f J(\xi) + \omega K(\omega) \right) \]

\[ + \dot{z}_z \left( f^5 J(\xi) + n^5 K(\omega) \right) / 120 \left( f J(\xi) + \omega K(\omega) \right). \]  

(47)

We compute, using (11) and (12)

\[ f^5 J(\xi) + \omega^5 K(\omega) = (f^6 + \omega^6 + (f^4 - \omega^4) 2 \Omega \sin^2 L) / 2 \Omega \cos L, \]

\[ = (f^2 + \omega^2) \left( (f^2 - \omega^2)^2 + (f^2 - \omega^2)^2 \Omega \sin^2 L / 2 \Omega \cos L. \]  

(48)

Using (31) and (32), (48) becomes

\[ (f^2 + \omega^2) \left( 8 \Omega^2 \cos L \left( 2 \Omega^2 - n \right) + n^2 \right) / 2 \Omega \cos L. \]  

(49)
Using (49), (27), (34) and (31), (49) becomes

\[ \dot{\chi}_s \Omega^2 \cos L \sin L (-4 \Omega^2 + n) / 30 \]
\[ + \dot{z}_s (8 \Omega^2 \cos^2 L (2 \Omega^2 - n) + n^2) / 120. \]  

(50)

And (50) is equal to

\[ ( -2/15 ) \Omega^4 \cos L \left( \dot{\chi}_s \sin L - \dot{z}_s \cos L \right) \]
\[ + n \Omega^2 \cos L \left( \left( \dot{\chi}_s \sin L - 2 \dot{z}_s \cos L \right) / 30 \right) + \dot{z}_s n^2 / 120, \]  

(51)

which is the coefficient of \( t^5 \) in (13).
RESOLUTIONS

1. The attendants of the Workshop express their sincere thanks to the French National Committee for Geodesy and Geophysics, to the Institut de Physique du Globe and to the Director of the BGI, Dr. BALMINO, and his staff for their support in organizing the meeting.

2. The Workshop expresses its sincere thanks to the Director of BIPM for providing a location for the absolute apparatuses operated during the meeting. Special thanks are due to Dr. SAKUMA and his colleagues for the assistance in making the measurements.

3. Considering the importance of high-precision tidal corrections to absolute and relative gravimetry, it is recommended that a standard tidal correction be adopted where the problem of the permanent tide is resolved in a manner satisfactory to all relevant disciplines.

4. Recognizing the role of highly precise gravity measurements in research related to crustal dynamics, earthquake occurrence, etc., and considering the necessity of improving their precision to carry out such studies the Workshop fully endorses the international D-meter campaign as designed and planned during its meeting.

5. Recognizing the continuing need for close cooperation of gravimetrists, the Workshop recommends that a request be forwarded to the Soviet Geophysical Committee to organize a meeting of specialists in high-precision relative and absolute gravity measurements in 1984.

6. All relative gravity measurements carried out in connection with absolute measurements during the International Campaign (Oct. 1981) should be processed by the observers according to the rules established by Prof. BOULANGER. For tidal corrections the Cartwright-Edden model should be used with a provisional gravimetric factor of 1.16 and a provisional phase lag zero. The data processed in this way should be sent to Prof. GROten no later than December 1, 1981. He will collect them and will send the combined results to all participants of the campaign no later than January 1, 1982.
7. **Recognizing** the need to publish the results of this Workshop and the inter-comparison of absolute gravity apparatuses which has taken place simultaneously the Workshop **recommends** that

(a) all manuscripts be published in the Bulletin d'Information as soon as possible (authors are requested to send them to Dr. BALMINO, by Dec. 1, 1981),

(b) results of comparing absolute gravity apparatuses be published in Bulletin d'Information,

(c) all countries possessing absolute apparatuses be asked to send a description of their equipment and measurement procedures, including a detailed discussion of systematic errors, to Dr. Ralph HUDSON, editor of Metrologia at B.I.P.M. by Jan. 1, 1982 for publication in that journal.

8. **Recognizing** that IAG adopted at the Canberra IUGG General Assembly a resolution calling for the presentation of gravity results on a tide-free for research in physical geodesy, and further **recognizing** that geophysicists, astronomers and oceanographers have a need for gravity data based on an earth model which includes the permanent tide, and **considering** the obligation of the International Gravity Commission through the Bureau Gravimétrique International to provide gravity data in a form suitable for all users,

the Workshop **recommends** that

(a) BGI provide gravity values to the users in either system,

(b) the Director of the BGI publish a notice in the Bulletin d'Information to this effect, and

(c) the Director of BGI prepare a short note, to be included with completed requests for data, describing the two tidal reduction systems, and further **recommends**

(d) that all agencies submitting gravity data to the BGI clearly state which kind of tidal reduction has been employed.

8. **Recognizing** the advisability of intercomparing absolute apparatuses on a regular basis,

the Workshop **recommends** that the Director of B.I.P.M. be approached by the International Gravity Commission to arrange a repetition of this year's experiment.
PART III

LIST OF PUBLICATIONS DEALING WITH GRAVITY MATTERS RECEIVED AT B.G.I.

(Bull. 48 continued)
# Table of Subject Indexes

<table>
<thead>
<tr>
<th>ALT.</th>
<th>ALTIMETRIE</th>
</tr>
</thead>
<tbody>
<tr>
<td>ANØ</td>
<td>INTERPRETATION DES ANOMALIES</td>
</tr>
<tr>
<td>BIB</td>
<td>BIBLIOGRAPHIE</td>
</tr>
<tr>
<td>DAT</td>
<td>DATA MANAGEMENT</td>
</tr>
<tr>
<td>DEV</td>
<td>DEVIATION DE LA VERTICALE</td>
</tr>
<tr>
<td>ECO</td>
<td>MOUVEMENT DE L'ECORCE TERRESTRE</td>
</tr>
<tr>
<td>EL</td>
<td>ELLIPSOIDE</td>
</tr>
<tr>
<td>ET</td>
<td>EITALONNAGE</td>
</tr>
<tr>
<td>ETA</td>
<td>EITALONNAGE AMERIQUE</td>
</tr>
<tr>
<td>ETE</td>
<td>EITALONNAGE EUROPEEN</td>
</tr>
<tr>
<td>ETP</td>
<td>EITALONNAGE ASIE/PACIFIQUE OUEST</td>
</tr>
<tr>
<td>GEN</td>
<td>GENERALITES</td>
</tr>
<tr>
<td>GEO</td>
<td>GEOPHYSIQUE GENERALE</td>
</tr>
<tr>
<td>GER</td>
<td>GEOIDE RESULTATS</td>
</tr>
<tr>
<td>GET</td>
<td>GEOIDE THEORIE</td>
</tr>
<tr>
<td>GN</td>
<td>GRAVITE ET NIVELLEMENT</td>
</tr>
<tr>
<td>GRA</td>
<td>GRAVITATION</td>
</tr>
<tr>
<td>GRD</td>
<td>GRADIENT VERTICAL</td>
</tr>
<tr>
<td>GVM</td>
<td>GRAVIMETRE</td>
</tr>
<tr>
<td>INT</td>
<td>PHYSIQUE DE L'INTERIEUR DE LA TERRE</td>
</tr>
<tr>
<td>ISO</td>
<td>ISOSTASIE</td>
</tr>
<tr>
<td>MA</td>
<td>MESURES AIR</td>
</tr>
<tr>
<td>MAB</td>
<td>MESURES ABSOLUES</td>
</tr>
<tr>
<td>MIEG</td>
<td>MESURES MER GENERALITES</td>
</tr>
<tr>
<td>MMR</td>
<td>MESURES MER RESULTATS</td>
</tr>
<tr>
<td>MF</td>
<td>MESURES PENDULAIRES</td>
</tr>
<tr>
<td>MT</td>
<td>MAREES TERRESTRES</td>
</tr>
<tr>
<td>NUM</td>
<td>CALCULATEURS ELECTRONIQUES</td>
</tr>
<tr>
<td>PEN</td>
<td>PENDULE</td>
</tr>
<tr>
<td>PMA</td>
<td>PESANTEUR &amp; MAGNETISME</td>
</tr>
<tr>
<td>PME</td>
<td>PESANTEUR &amp; METEOROLOGIE</td>
</tr>
<tr>
<td>PS</td>
<td>PESANTEUR &amp; SEISMICITE</td>
</tr>
<tr>
<td>RE</td>
<td>RESEAU EUROPEEN</td>
</tr>
<tr>
<td>RED</td>
<td>METHODES DE REDUCTION</td>
</tr>
<tr>
<td>RIC</td>
<td>RESEAU INTERNATIONAL COMPENSATION</td>
</tr>
</tbody>
</table>
RIP

SAT.

SEL

STA

THP

VMG

VS

WDC

WDC A

WDC B

RESEAU INTERNATIONAL 1er ORDRE

SATELLITES

SELENODESIE

STATISTIQUE

THEORIE DU POTENTIEL

VALEURS MOYENNES DE LA GRAVITE

VARIATION SECULAIRE

WORLD DATA CENTERS GENERAL

U.S.A. (BOULDER)

U.R.S.S. (MOSCOU)
## LIST OF COUNTRY CODES

<table>
<thead>
<tr>
<th>Africa</th>
<th>North America</th>
<th>Oceania</th>
<th>Europe</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 AFGHANISTAN</td>
<td>101 CANADA</td>
<td>401 AUSTRALIA</td>
<td>501 ALBANIA</td>
</tr>
<tr>
<td>2 ALGERIA</td>
<td>102 U.S.A.</td>
<td>402 FIJI</td>
<td>502 ANDorra</td>
</tr>
<tr>
<td>3 ALBANIA</td>
<td></td>
<td>403 INDIA</td>
<td>503 AUSTRIA</td>
</tr>
<tr>
<td>4 COMORO</td>
<td></td>
<td>404 IRAQ</td>
<td>504 BELGIUM</td>
</tr>
<tr>
<td>5 CAMEROON</td>
<td></td>
<td>405 IRAQI PALM BEAR</td>
<td>505 BULGARIA</td>
</tr>
<tr>
<td>6 CAPE VERDE</td>
<td></td>
<td>406 JAPAN</td>
<td>506 CYPRUS</td>
</tr>
<tr>
<td>7 CENTRAL AFRICAN REP.</td>
<td></td>
<td>407 KOREA</td>
<td>507 CZECHOSLOVAKIA</td>
</tr>
<tr>
<td>8 CHAD</td>
<td></td>
<td>408 LAOS</td>
<td>508 DENMARK</td>
</tr>
<tr>
<td>9 CHINA</td>
<td></td>
<td>409 LATVIA</td>
<td>509 FINLAND</td>
</tr>
<tr>
<td>10 COMORO</td>
<td></td>
<td>410 LIECHTENSTEIN</td>
<td>510 FRANCE</td>
</tr>
<tr>
<td>11 CONGO</td>
<td></td>
<td>411 LUXEMBOURG</td>
<td>511 GERMANY</td>
</tr>
<tr>
<td>12 DJIBOUTI</td>
<td></td>
<td>412 MALTA</td>
<td>512 GERMANY WDT</td>
</tr>
<tr>
<td>13 EGYPT</td>
<td></td>
<td>413 MEXICO</td>
<td>513 GREECE</td>
</tr>
<tr>
<td>14 EQUATORIAL GUINEA</td>
<td></td>
<td>414 MONGOLIA</td>
<td>514 HUNGARY</td>
</tr>
<tr>
<td>15 ETHIOPIA</td>
<td></td>
<td>415 MONACO</td>
<td>515 ICELAND</td>
</tr>
<tr>
<td>16 EQUATORIAL GUINEA</td>
<td></td>
<td>416 NAMIBIA</td>
<td>516 IRELAND</td>
</tr>
<tr>
<td>17 GHANA</td>
<td></td>
<td>417 NAMIBIA</td>
<td>517 ITALY</td>
</tr>
<tr>
<td>18 GAMBIA</td>
<td></td>
<td>418 NAMIBIA</td>
<td>518 LIECHTENSTEIN</td>
</tr>
<tr>
<td>19 Gambia</td>
<td></td>
<td>419 NEDERLANDS</td>
<td>519 LUXEMBOURG</td>
</tr>
<tr>
<td>20 GUINEA-CONACAO</td>
<td></td>
<td>420 NETHERLANDS</td>
<td>520 MALTA</td>
</tr>
<tr>
<td>21 IVORY COAST</td>
<td></td>
<td>421 NIGERIA</td>
<td>521 MONACO</td>
</tr>
<tr>
<td>22 KENYA</td>
<td></td>
<td>422 NIGERIA</td>
<td>522 NETHERLANDS</td>
</tr>
<tr>
<td>23 LESOTHO</td>
<td></td>
<td>423 NIGERIA</td>
<td>523 NORWAY</td>
</tr>
<tr>
<td>24 LITHUANIA</td>
<td></td>
<td>424 NIGERIA</td>
<td>524 POLAND</td>
</tr>
<tr>
<td>25 LITHUANIA</td>
<td></td>
<td>425 NIGERIA</td>
<td>525 PORTUGAL</td>
</tr>
<tr>
<td>26 MALAWI</td>
<td></td>
<td>426 NIGERIA</td>
<td>526 ROMANIA</td>
</tr>
<tr>
<td>27 MALI</td>
<td></td>
<td>427 NIGERIA</td>
<td>527 SAO TOME-PRINCIPE</td>
</tr>
<tr>
<td>28 MAURITIUS</td>
<td></td>
<td>428 NIGERIA</td>
<td>528 SARAMAKI</td>
</tr>
<tr>
<td>29 MAURITIUS</td>
<td></td>
<td>429 NIGERIA</td>
<td>529 SPAIN</td>
</tr>
<tr>
<td>30 MEXICO</td>
<td></td>
<td>430 NIGERIA</td>
<td>530 SWEDEN</td>
</tr>
<tr>
<td>31 MOZAMBIQUE</td>
<td></td>
<td>431 NIGERIA</td>
<td>531 SWITZERLAND</td>
</tr>
<tr>
<td>32 NAMIBIA</td>
<td></td>
<td>432 NIGERIA</td>
<td>532 UNITED KINGDOM</td>
</tr>
<tr>
<td>33 NAMIBIA</td>
<td></td>
<td>433 NIGERIA</td>
<td>533 YUGOSLAVIA</td>
</tr>
<tr>
<td>34 NIGERIA</td>
<td></td>
<td>434 NIGERIA</td>
<td></td>
</tr>
<tr>
<td>35 NIGERIA</td>
<td></td>
<td>435 NIGERIA</td>
<td></td>
</tr>
<tr>
<td>36 NIGERIA</td>
<td></td>
<td>436 NIGERIA</td>
<td></td>
</tr>
<tr>
<td>37 SAO TOME-PRINCIPE</td>
<td></td>
<td>437 NIGERIA</td>
<td></td>
</tr>
<tr>
<td>38 SENEGAL</td>
<td></td>
<td>438 NIGERIA</td>
<td></td>
</tr>
<tr>
<td>39 SEYCHELLES</td>
<td></td>
<td>439 NIGERIA</td>
<td></td>
</tr>
<tr>
<td>40 SIERRA LEONE</td>
<td></td>
<td>440 NIGERIA</td>
<td></td>
</tr>
<tr>
<td>41 SOMALIA</td>
<td></td>
<td>441 NIGERIA</td>
<td></td>
</tr>
<tr>
<td>42 SOUTH AFRICA</td>
<td></td>
<td>442 NIGERIA</td>
<td></td>
</tr>
<tr>
<td>43 SUAZAN</td>
<td></td>
<td>443 NIGERIA</td>
<td></td>
</tr>
<tr>
<td>44 SWAZILAND</td>
<td></td>
<td>444 NIGERIA</td>
<td></td>
</tr>
<tr>
<td>45 TANZANIA</td>
<td></td>
<td>445 NIGERIA</td>
<td></td>
</tr>
<tr>
<td>46 TOGO</td>
<td></td>
<td>446 NIGERIA</td>
<td></td>
</tr>
<tr>
<td>47 TUNISIA</td>
<td></td>
<td>447 NIGERIA</td>
<td></td>
</tr>
<tr>
<td>48 UGANDA</td>
<td></td>
<td>448 NIGERIA</td>
<td></td>
</tr>
<tr>
<td>49 UPPER VOLTA</td>
<td></td>
<td>449 NIGERIA</td>
<td></td>
</tr>
<tr>
<td>50 ZAIK</td>
<td></td>
<td>450 NIGERIA</td>
<td></td>
</tr>
<tr>
<td>51 ZAMBIA</td>
<td></td>
<td>451 NIGERIA</td>
<td></td>
</tr>
<tr>
<td>52 ZIMBABWE</td>
<td></td>
<td>452 NIGERIA</td>
<td></td>
</tr>
</tbody>
</table>

During 1976, the Bureau of Mineral Resources made a geophysical survey at the Mount Turner porphyry copper prospect, near Georgetown, Queensland. The objectives of the survey were to assist the geological evaluation of the porphyry copper prospect and to study the applications for geophysics in exploration for porphyry copper orebodies. The prospect occurs in an alteration system within a pluton of Proterozoic Forsyth Granite, which contains enclaves of Robertson River Metamorphics. Vertical electrical soundings were made to map the chargeability and resistivity of the un weathered bedrock. Resistivity and chargeability zones are identified and appear to map out areas of different alteration types. Gamma-ray spectrometry, total-count radioactivity, and magnetics did not contribute to the porphyry copper prospect.


An earthquake occurred on 2 June 1979 near the small town of Cadoux, Western Australia. Only one person was injured, but the cost of damage in the town and surrounding district could exceed $1.5 million. Preliminary results show that the earthquake had a Richter magnitude of 6.2 and accented at 09h 48m 01s UT at latitude 30° 83' S, longitude 117° 15' E, and at a depth of 15 km. It is the third earthquake of magnitude 6 or greater to occur in the southwest scismatic zone in eleven years. The maximum Modified Mercalli intensity observed was IX. The surface of the Earth fractured in a zone 14 km long. Three scarpas were formed, the largest was 8 km long and ran northerly with overthrusting up to 1.1 m from the west, and vertical uplift up to 0.6 m. Some right-hand strike-slip occurred. The general direction of movement (70°-80°) conformed roughly with the direction of the axis of maximum stress measured in the area in 1976. The two smaller faults (2 km and 5 km long) ran about southeast and the area between them was raised by up to 0.5 m and left-lateral motion reached 0.6 m.

A map of the fracture zone is located on the other side of this card. Figures 5, 6, 7, 8, 9, and 13 are reproduced by courtesy of West Australian Newspapers Limited.


Software has been developed for storing any amount of satellite altimeter data on tape and disk devices with the ability to select parts of existing data sets and form new ones as required. The same data set structure is employed for storage and applications. Users are encouraged to run the maintenance software to tailor altimeter data sets to their specific needs. These features were designed to provide sufficient data manipulation capability for research without the extensive developmental effort needed for a true data base. The system is written in IBM FORTRAN. Instructions for the user are not provided because they are subject to change.


The computation of geopotential differences from levelling data requires the knowledge of gravity values with a spacing Δs between the points of observation depending on the gravity prediction root mean square error 1 and the height variations ΔH along the levelling line...

Various gravity prediction techniques are briefly reviewed and compared.


102 - CARRUTHERS, G. - "The importance of mineral and energy inventories".

102 - CARRUTHERS, G. - "General guidelines for national inventories of mineral resources".


This report summarizes results of Doppler point positioning data obtained during 2 intervals: June 1975, and February 1978, at two separate sites at Onsala Space Observatory (OSO). The computations were carried out at Upplands, 1965-72 system, and intercomparision with several different results is made. It is shown that a discrepancy of \( \pm 10 \) meters in the Z component of the Doppler for OSO-1 exists in earlier reported results for the 1975 data.

Earth tidal data acquired during May 1979 at OSO are analysed. Although the duration of data is short, evaluation of the results with theoretical values has been satisfactory. Anomalous amplitudes and phase lags of \( M_s \) and \( M_2 \) tidal waves are found, which indicates periodic systematic ocean loading effects of the order of 1 cm in vertical displacement occur at OSO.

During the course of preparation of this report an unsatisfactory lack of agreement was found in the geodetic material made available for this study. Likewise, the interval of earth tidal recording available was too short for a firm analysis to be made.

GEO - NAKAI, Y., et al. - "Explanational text of geological map of Kjuka and Telen Antarctica".

Kjuka and Telen are located at 69° 34' - 69° 40' S and 39° 37' - 39° 50' E in Saya Coast which is the eastern coast of Lützen-Holm Bay, Antarctie. The Telen Glacier about 4.5 km in width separate the Telen area from the Kjuka area.

The topographical map "Kjuka and Telen" on a scale of 1:25000 was published in March 1975 by the Geographical Survey Institute of Japan; the map was compiled from air-photographs taken on January 21, 1962 and January 20, 1969 with ground air-photos control points established by the 14th Japanese Antarctic Research Expedition (JARE-14).

The geological survey in Kjuka was carried out in 1973 by Terumi Ishikawa, a member of JARE-13. The geology of Telen was surveyed by Yutaka Nakai, Takashi Iwano and Shuichi Yoshikura, members of JARE-19, between January 31 and February 1, 1978.

GEO - SUZUKI, H., MORIHAKI, K. - "Explanational text of geological map of Cape Omega Antartica".

8. ANDERSEN, A.J. - "Pennesian uplift secular variations of gravity and determination of changes of density within the asthenosphere".

Studies of disturbing effects upon the measurements of gravity along a central microgravity profile in Pennesia are investigated. The largest uplift gravity gradient differences expected along the profile occur in the mountainous region separating Norway and Sweden. These differences amount to 0.4 µgal/year/100 km.

Disturbing quantities which are investigated across the profile are ocean loading tidal variations, ground water variations, snow cover and sea level water height changes, and the effects of regional meteorological pressure differences across Pennesia. Very stringent control of all of the above disturbing quantities must be made in order for a significant secular gravity gradient uplift model to be obtained on a reasonable time scale. Furthermore the determination of short term instrumental drift characteristics need to be improved by an order of magnitude over that obtained in previous experiments.

MF - SCHEINBAUM, H.G. - "A drift correcting apparatus for tidal gravimeters".

An apparatus has been built for automatic readjustment of a Baront modified ASKAT-GS II recording gravimeter in order to keep the measuring device within a tolerable limit around its centre position. This device marks one step towards permanent installation of recording earth-tide stations that operate without periodic maintenance.
squares prediction and determination of variances in geodetic networks comprising different types of observations.


Mixed models contain parameters defined as fixed quantities and as random variables with zero expectations. To estimate the parameters best linear unbiased estimates in the Gaussian-Markov model are applied for the fixed parameters and in the generalized Gaussian-Markov model for the random parameters after introducing an approximate mixed model. It is shown that these estimates are identical with the estimates in the mixed model by means of the least squares adjustment and the maximum likelihood method.


9 Earth and some aspects of the definition of gravity

A brief review of crustal movements within the Pennscandia shield is given. Results from post-glacial studies, projects for measuring active fault regions, and dynamic ocean loading experiments are presented. The 1979 Scandinavian Doppler Campaign (SCANDOC 79) Network is discussed. This network includes Doppler translocation baseline determination of future VLBI baselines to be measured in Scandinavia. Intercomparison of earlier Doppler translocation measurements with a high precision terrestrial geodetic baseline in Scandinavia has yielded internal agreement of 6 cm over 887 km. This indicates a precision of better than 1 part in 10^9 may be obtainable with these techniques.


A new method for determining the early history of the Earth-Moon system is described. Called the study of lunar paleolites, it describes a method for explaining features of the remnant lunar gravity field, and the generation of the lunar mascons. A method for the determination of Earth-Moon distances compared with the radiometric ages of the maria is developed. It is shown that the Moon underwent strong anomalous gravitational tidal forces, for a duration t < 10^9 years, prior to the formation of the mascon surfaces. As these tidal forces had not been present at the time of the formation of the Moon, this shows that the Moon could not have been formed in orbit about the Earth.


10 A brief review of crustal movements within the Pennscandia shield is given. Results from post-glacial studies, projects for measuring active fault regions, and dynamic ocean loading experiments are presented. The 1979 Scandinavian Doppler Campaign (SCANDOC 79) Network is discussed. This network includes Doppler translocation baseline determination of future VLBI baselines to be measured in Scandinavia. Intercomparison of earlier Doppler translocation measurements with a high precision terrestrial geodetic baseline in Scandinavia has yielded internal agreement of 6 cm over 887 km. This indicates a precision of better than 1 part in 10^9 may be obtainable with these techniques.


In this paper an attempt is made to deduce a general formula for the time-independent part of the tidal effect and its role in tidal corrections to geodetic observations.


A dialogue on collocation


A brief review of crustal movements within the Pennscandia shield is given. Results from post-glacial studies, projects for measuring active fault regions, and dynamic ocean loading experiments are presented. The 1979 Scandinavian Doppler Campaign (SCANDOC 79) Network is discussed. This network includes Doppler translocation baseline determination of future VLBI baselines to be measured in Scandinavia. Intercomparison of earlier Doppler translocation measurements with a high precision terrestrial geodetic baseline in Scandinavia has yielded internal agreement of 6 cm over 887 km. This indicates a precision of better than 1 part in 10^9 may be obtainable with these techniques.


A dialogue on collocation
Comparisons of gravimetric and astrogeodetic deflections of the vertical in the regional domain indicate that the former are affected by position-dependent systematic errors, even after orientation onto the Australian Geodetic Datum. These are probably due to errors in the predicted mean anomalies for gravimetrically surveyed oceanic regions to the east, south and west of the continent. Deflection component residuals (astrogeodetic minus oriented gravimetric) at 83 control stations are made available in a set of observation equations, based on the Vening Meinesz equations, from which pseudocorrections to the mean anomalies for a set of arbitrarily selected surface elements are computed. These pseudocorrections compensate for prediction errors in much larger unsurveyed regions. Their effects on individual deflection components are calculated using the Vening Meinesz equations. Statistical tests indicate that pseudocorrections computed for 4400 stations, which are based on 83 offshore areas and 50 smaller elements in unsurveyed areas, produce corrections to the gravimetric deflections which are consistent with measured distributions on the earth.

The geodetic boundary value problem (p.b.v.p.) is a free boundary value problem for the Laplace operator; however, under suitable change of coordinates, it can be transformed into a fixed boundary one. Using a general coordinate choice problem arising are two particular cases are more clearly analyzed, namely the gravity space approach to the intrinsic coordinates (Marussi) approach.

In the theory of gravitation, usually a potential function is introduced to describe the gravity field. This has the advantage that Green's integral representation of potential theory can be used in a straightforward manner. However, the introduction of a scalar potential also has some disadvantages. Although the potential can be associated with the work done when moving around a point mass in a gravitational field, it still is a somewhat artificial quantity that cannot directly and unambiguously be measured. This is in contrast with the gravitational field strength. In the presence paper we investigate the possibility of avoiding the introduction of a gravitational potential and concentrating on the gravitational field vector itself. In that case, appropriate vector integral representations of the Green type have to be derived for the gravitational field. It turns out that these vector integral representations are also suited for describing the gravity field of the earth.

A spherical harmonic expansion of the gravity field up to degree and order 200 was carried out. Free air anomaly data over Canada ("x" x " block averages) with a range of 211.1 mgal were used for testing. A low degree expansion (N = 30) produced a map with a range of 63.6 mgal with contour patterns that could hardly be correlated with the original data contours. A high order expansion (N = 200) on the other hand resulted in a map with a range of 199.8 mgal which quite faithfully reproduced the original including local variations. Test computations verify that by monitoring the BGS values and the range of the expansion it is possible to arrive at an optimum degree of expansion for a given data set. It was also verified by the computations, that since the computed expansions essentially have a zero value outside the domain of the input, it is possible to combine the results of separate non-overlapping expansions.
The gravity map of the Adrar des Iforas (Mali) completes the survey of the eastern margin of the West African craton. The interpretation of the gravity data is based on a confrontation with the geological and aeromagnetic maps. The main features are:

- the virgation of Pan-African structures;
- the suture is characterized by the tectonic emplacement of ultrabasic and basic massifs with thrusting towards the craton and NNW-SSE transversal faults;
- important positive anomalies are associated with late Pan-African granitoides;
- within the craton a large positive anomaly is related to the Gourma aureole.

The quantitative interpretation using the inverse approach linear programming and statistical analysis allows one to estimate the form, density and depth of some structures.

It is suggested that during collision the shape of the margin of the craton which corresponds to the suture as defined by the gravity pattern has had a controlling influence on many tectonic features within the Pan-African mobile belt.

A gravity survey of Reunion Island shows a NW-SE anomaly joining the positive anomaly associated with Piton des Nigros to another positive anomaly on the east coast. The Piton de la Fournaise volcano is marked by short wavelength negative anomalies.


Regional airborne magnetic and gravity data and field observations have been used to define four geophysical domains in the Lachlan Fold Belt, South Wales. Each has a different pattern of anomaly trend and amplitude. Three domains correspond with provinces containing rocks of similar lithology and age; the other corresponds with the Darling Basin. The magnetic data highlight minor variation in magnetic abundance, chiefly in igneous rocks, and, across the region, reflect slight differences in magma composition, structural trends, and possibly also style of volcanism and sedimentation. The gravity data also delineate the same structural trends and some major lithological variations. The boundary of the Darling Basin is gradational. The other domain boundaries are sharp, but with no evidence of major faulting.

Within the geophysical domains a classification of magnetic anomalies by length, width, and amplitude appears capable of distinguishing between the various sources, thereby providing a useful mapping tool in regions without surficial cover. Circular anomalies of largest extent occur over granite, the smallest over small pipes and veins, and the intermediate size over basic and ultrabasic rocks. Elliptical anomalies are associated with magnetic granitoides and some basic and ultrabasic bodies. Sources of narrow anomalies include basic dykes, steeply tilted Plaeeocratic lavas, ignimbrites, and serpentinites. Sources of complex zones include Tertiary basin fill flows, piles of basic and intermediate lavas surrounding small intrusions, and some inhomogeneous granitoides.

The regional gravity data reflect features with very large dimensions or large density contrasts. A major north-northeast trending Bouguer anomaly low corresponds with the Eastern Highlands, a region of high relief, thick crust, and extensive granitoides. A narrower low corresponds to another belt of granitoids trending northeast-northwest from Bobrock to Cobarr. Small-scale gravity features include the Coolang Serpentinite and its associated basic rocks, the Mid-Sillurian to Mid-devonian Hill End Trough, and numerous regions of Late Devonian quartzose sedimentary rocks.


A geodetic earth reference model is defined as a self-gravitating body of given mass and rotational rate whose surface is an equipotential ellipsoid of revolution of specified dimensions. Over the course of this century, the International Association of Geodesy has sanctioned three such models as recommended standards for both scientific and practical applications. The most recent model was approved in December 1976, which in turn supplanted another originally adopted four decades earlier.
The Goddard Earth Model 10C and a model based upon Rapp’s potential coefficients solve complete to degree and order 100 are compared with doppler-derived geoid undulations, ESKAT-A altimeter values and point free-air gravity anomalies in the Norwegian-Greenland Sea and in the coastal areas. An agreement between observed and computed gravity anomalies at the 22 mgal level (compared to a r.m.s. of the observed values of 7.46 mgal). A comparable result is found for deflections of the vertical in the Scandinavian area.

It is pointed out, that collocation is a suitable method for making local corrections to or improvements of a global gravity field model. This is illustrated through the computation of an improved geoid for the area bounded by 66°56’-69°55’, 7°55’-16°55’.

Finally the geoid calculated based on Rapp’s coefficients is compared with undulations derived solely from the isostratically compensated sea-bottom topography. This comparison exposes geophysical information, which otherwise is not clearly seen.

The general problem of determining the figure of the earth leads to the solution of the geodetic boundary value problem. By its discrete approximations we obtain the discrete disturbing potential that maintains all properties of the original problem. Thus, the discrete approximation of the disturbing potential can be used in studying the behaviour of the earth’s gravity field outside the disturbing masses. The deflections of the vertical are one of the quantities describing the behaviour of the earth’s gravity field. A method for their computation from the discrete solution of the geodetic boundary value problem is put forth and estimates for its accuracy are given.

Fourier transformation of the theoretical gravity effect due to a two-dimensional asymmetrical triangular prism has been derived. Evaluation of the parameters from the analysis of the Fourier Spectra has been outlined.

Fourier transformation of gravity and magnetic anomalies from space to the frequency domain provides a ready method for source depth estimation since progressively deeper sources are indicated by the lower frequency components of an anomaly. Two examples are presented here to demonstrate the efficacy of this approach. One example pertains to the interpretation of vertical intensity magnetic anomalies over a layered ultramafic body for estimating the thickness of relatively non-magnetic layer in it and map the dispositions of the layers. The other example is the estimation of crustal thickness in the Indian region from the Bouguer anomaly map. The spectral method is shown, by these two examples, to provide a rapid and elegant tool for the source depth estimation for magnetic and gravity data.

The interpretation of the gravity and magnetic fields from inclined dikes has been studied with artificial data contaminated by various noise components: base level, linear trend, and random noise. A Gaussian window was applied to the data prior to transformation to reduce the influence of noise as demonstrated by an analysis of the horizontal cylinder. The case of the dike is more complicated due to the fact that its spectrum has a number of zeros at wavenumbers which are inversely related to the width of the dike. Around these wavenumbers, especially the random noise distorts the spectrum making interpretation ambiguous.

Detailed gravity measurements recently carried out on the Corner glacier, Switzerland, are used to determine the variations of thickness across the glacier ice. The Corner Glacier was chosen as a test site because seismic control was available. The glacier ice at a profile near the Monte Rosa massif is associated with a relative gravity low of about 23 mgal. Model calculations yield a corresponding ice thickness of about 400 m at the central part of the profile. A comparison of the derived residual gravity anomaly with the calculated effect of a 3-D ice model based on seismic information is made. It is shown that the regional field determined for the Corner glacier is appropriate and gives the correct residual anomaly associated with the glacier ice. Therefore, the proposed gravity technique for determining variations of the thickness of glacier ice appears to be a valuable and rather inexpensive method for surveying glaciers.

An interpretation of potential fields from inclined dikes in the wavenumber domain was carried out on the Corner glacier, Switzerland, and a method for determining the thickness of glacier ice was proposed. The Corner Glacier was chosen as a test site because seismic control was available. The glacier ice at a profile near the Monte Rosa massif is associated with a relative gravity low of about 23 mgal. Model calculations yield a corresponding ice thickness of about 400 m at the central part of the profile. A comparison of the derived residual gravity anomaly with the calculated effect of a 3-D ice model based on seismic information is made. It is shown that the regional field determined for the Corner glacier is appropriate and gives the correct residual anomaly associated with the glacier ice. Therefore, the proposed gravity technique for determining variations of the thickness of glacier ice appears to be a valuable and rather inexpensive method for surveying glaciers.

A mass distribution which remains constant perpendicular to a diameter, within the walls of a vertical cylinder, is defined as a bounded two-dimensional mass distribution. Assumption of such a distribution may be more realistic than the assumption of an unbounded two-dimensional distribution in many situations. The gravity effect of such a distribution is comparable with the effect calculated by conventional three-dimensional methods of reduction. The effect of sphericity of the earth may be allowed as an additional correction. For the calculation of the gravitational effect of this distribution, a formula for the attraction of a cylindrical strip, at a point on the axis, is derived. The effect is compared with that of a two-dimensional distribution. To calculate the effect of sphericity, a formula for the attraction of a spherical cap is derived and this attraction is compared with that of a flat disc. The proposed distribution and the correction for sphericity are particularly applicable in marine gravity reductions.

A subroutine package for processing large, sparse least-squares problems has been compiled to make it relatively easy for application programmers to use several existing subroutines for forming and solving very large, sparse sets of normal equations. The package provides a method for reordering, solving, and computing a portion of the inverse for sparse matrices, including the step of building the normal equations from the observation equations.

Technical Memorandum, NOS NGS 29, 1-18, Jan 1981.
The two sections of the package include (1) reordering the normal equations and (2) building the normal equations. It then reduces, solves, and computes the inverse. Although the two sections can be used independently, they were designed to be used together. Implementing each section involves three basic steps: (1) initialize some work space, (2) pass the observation equations to the system, and (3) call the function for the needed operation. The number of parameters passed in each call has been kept to a minimum to make the procedure easy to understand and use. A simple example demonstrates how quickly the package can be implemented.

101 - YASUKUCHI, S.A. - "Investigation of the stability of the mekometer"

The Geodetic Survey has used a Kern M3000 Mekometer since 1976 to precisely measure baselines designed for the calibration of other electronic distance measuring instruments. For this purpose it is necessary to have a detailed knowledge of the behaviour of the Mekometer under varying conditions. Error sources, field calibration and laboratory calibration procedures are described. Calibrations in the field and in the lab are compared. The service record of the instrument is reviewed. It is recommended that lab frequency be continued and that an altitude correction be applied for high altitude baselines.

101 - YAMOSI, S. - "Three dimensional positioning with MX 1502 satellite surveyor"

Two MX 1502 Satellite Surveyors have been utilized on an experimental basis simultaneously operating with two CGA-722B Satellite Doppler Survey units set up at stations 102 to 108 in 1978. The paper describes the experiment and results obtained from the first operation of the MX 1502 by the Geodetic Survey of Canada. The MX 1502 is a modern microcomputerized satellite surveying receiver built in one unit, capable of automatically producing a three-dimensional positioning almost immediately after the end of a satellite pass, using On-Board (broadcast) Ephemeris. The CGA-722B is an early (1973) receiver model presently in use by the Geodetic Survey of Canada. It has since been superseded by the CGA-751 which was not available at the time of the test.

101 - WILKINSON, G.A. - "Covariance analysis for a local level inertial survey system"

Inertial survey systems use real-time optimal estimation techniques (Kalman filtering) to control time-dependent errors inherent in the system. Space and time limitations of the on-board computer make it necessary to use suboptimal filters based on simplified models derived from a larger, more complex model that takes into account all known effects. The complex nature of error propagation in these systems makes it necessary to use covariance analysis techniques for evaluation of proposed filter designs and investigation of the errors of individual error sources.

Theory and equations necessary for the analysis of covariance propagation are presented. A computer program has been written to generate simulated error covariance histories of suboptimal filter designs in inertial survey systems. These histories can be analyzed to evaluate the performance of the filter. The user defines the model to be used and the traverse configuration to be simulated. Results from sample simulations for the two-dimensional navigation problem are given.

101 - O'BRIEN, L.J. - "The combination of inertial and doppler systems for secondary surveys"

Survey data required to control mapping of the Canadian landmass at a scale of 1:50,000 is being accumulated rapidly through the employment of modern methods of data collection and reduction. An Inertial Survey System (ISS) is described and operational factors are summarized on experience gained since 1975. The establishment of a grid of horizontal control points by satellite Doppler methods at optimum locations and baselines for subsequent interpolation by the ISS is discussed. Cost comparisons with more conventional survey methods are cited.

101 - SWANSON, H.T. - "Field experience with the Lambert "twin" tower at the Geodetic Survey of Canada"

A new survey tower, which combines the provision for precise angle measurement, characteristic of wooden steel structures, with the mobility and ease of assembly of lightweight aluminum structures, was tested under field conditions to establish its viability for use on first-order horizontal control surveys.

Constructive criticism from the user and cooperation from the manufacturer combined to produce a survey tower that can be used to advantage in the variety of field conditions encountered on conventional type control surveys.

101 - SCHWARZ, K.P., LACAPPELLE, G. - "Local characteristics of the gravity anomaly covariance function"

Using a data set of 260,000 gravity anomalies it is shown that common characteristics for a local covariance function exist in an area as large as Canada excluding the Rocky Mountains. After eliminating global features by referencing the data to the GEM-10 satellite solution, the shape of the covariance function is remarkably consistent from one sample area to the next. The determination of the essential parameters and the fitting of the covariance function are discussed in detail.

To test the reliability of the derived function, deflections of the vertical are estimated at about 230 stations where astrogeodetic data are available. Results show that the standard error obtained from the discrepancies is about 1" for each component and that the error covariance matrix of least-squares collocation reflects this accuracy remarkably well.

101 - FENTON, G. - "Inertial survey system"
151 Interdepartmental Coordinating Committee on Control Surveys Ottawa, Surveys and Mapping Branch, 178-192, 1979.

This presentation briefly describes what the Inertial Survey System is, how it works, and what it has produced to date. The ISS has been in service for four field seasons, beginning in 1976. Summaries are given for each year's production, including statistics on the number of stations surveyed and the estimated cost per station. The total of 3500 stations is considered in light of conventional methods in both cost per station and total time. The purchase price of the two complete systems owned by the Geodetic Survey is easily justified by the 2.5 million dollars saved over the past four years without considering the estimated six to eight extra field seasons needed to complete the task by conventional means. Finally, a brief account is given of the horizontal and vertical accuracy exhibited by the ISS on the 1979 Ottawa ISS test line.
The status of the North American levelling networks is described as well as some concerns and future needs in surveying, geodesy and earth sciences. Preparatory work including data automation, research of various related problems and relevelling campaigns is reviewed. The 100,000 km releveling program of the United States is described in detail. Past and future events related to the redefinition project are also given.

The effect of the departure of the vertical reference system used to derive free-air anomalies from a single equipotential surface in various geodetic applications is described. The departure of the vertical reference system used to calculate free-air gravity anomalies of the Canadian National Gravity Data Base from a single equipotential surface is investigated. Departure of height comparisons used to derive gravity anomaly at the recent and presumably better heights at over 1500 gravity anomalies coinciding with lake surfaces and distributed over large parts of Canada. Analysis of the results reveals statistically significant departures between different areas. The use of a least squares prediction method to estimate the departure at remaining gravity anomaly points is discussed and a related signal covariance function is derived from the numerical data. More sophisticated methods to estimate the above departures are proposed.

A method to predict deflections of the vertical in mountainous areas from a combination of topographic-isostatic and astrogeodetic data is described. The result for the Western Cordillera and the New Mexico area of the United States indicates that an accuracy of 270 to 275 can be obtained in these areas for the meridian and prime vertical deflection components when using readily available 1 km (North Cordillera) and 1 arcsec (New Mexico) topographic grids to evaluate the effect of the topography and its isostatic compensation in the immediate vicinity of computation points. When no astrogeodetic data is assumed to be available, an accuracy of about 37° is achieved on the basis of the topography only.

A brief historical review of first-order levelling networks in Canada is presented. The first continental Canadian adjustment of 1928 comprising over 30,000 km of levelling lines is described. The datum implied by this adjustment is still that currently used. The present status of first-order networks, now comprising over 100,000 km of levelling lines, is described. Important topics include topographic data coverage, vertical crustal movement and systematic errors affecting these networks are discussed. Plans and preparations for the forthcoming North American vertical redefinition are also treated.

Gravimetric and astrogeodetic (based on RML 9D Doppler system) deflection components at over 1600 stations of the Canadian geodetic framework and U.S. transcontinental traverse are compared in order to study the nonparallelism of the zero geodetic meridian plane of the RML 9D system with the zero astronomic meridian plane of the Bureau International de l’Haut (BID) and, secondly, possible biases between Canadian and U.S. astronomical longitude observations carried out by the Geodetic Survey of Canada and U.S. National Geodetic Survey respectively. Results pertaining...
to the first objective, using U.S. data only, are in agreement with those obtained previously by other investigators using different methods, namely that longitudes in the NWL system should be increased by about 0°8 to be compatible with the BII system. Results related to the second objective show there exists an average difference between Canadian and U.S. astronomic longitude observations of the order of 0°8. Further comparisons using data subsets in both countries reveal that the above difference varies from area to area; however, this result is based on the assumption that gravimetric deflections are themselves free of regional biases.


Any readjustment of a geodetic network also implies corrections for neighboring geodetic positions of lower order or classification. These corrections are usually obtained through local adjustments, or some other estimation procedures, depending on the geodetic observational information available. When only two sets of corresponding coordinates are given for a number of geodetic control stations, the positional differences can be evaluated using least-squares estimation. Because of geodetic considerations, some structural constraints need to be implemented in the estimation of a regional functional trend, followed by a residual interpolation procedure for local refinement. The use of complex variables has proven advantageous in formulating the estimation model. Practical results illustrate different characteristics of this proposed solution to the problems of geodetic control densification. The approach can also be adapted to other applications.


The definition of a geodetic horizontal datum involves different aspects which are closely interrelated. The consideration of terrestrial, astronomic and gravitational effects in an optically compatible reference system is required in network adjustments. The recent test adjustments of the Canadian horizontal framework have raised questions about systematic biases in the observational data. The different aspects of the solutions are reviewed in the sequel and some preliminary conclusions are drawn. A number of experiments and studies are currently being carried out in different places in preparation for the forthcoming readjustment of 1983. The principal motivation of this report is that another test adjustment of the Canadian horizontal framework be done before entering the data into the continental adjustment. Those test adjustment results would provide the final check on the observational data and prove useful in analyzing the continental adjustment results.


Satellite Doppler and inertial surveying methods continued to dominate the establishment of horizontal control in 1979. However, conventional terrestrial electronic distance measurement methods continued to provide accurate and useful control in urban areas and for some northern mapping projects. Precise levelling progress continued to be good, although the total distance covered was about 12% less than the previous year. Much less accurate, but extremely useful, elevations continue to be established by a variety of other methods such as the Ground Elevator Meter, airborne altimetry and trigonometric levelling.

Progress continued in the automation of the national control data bank, scheduled for completion in 1983. Although preparations for the redefinition of the new North American position datum continued as scheduled, the completion will now be 1986 because of delays in the U.S. effort. Approximately 50% of Geodetic Survey's national secondary control has now been evaluated, for integration by 1985 into the redefined primary system.

Research and development was conducted concerning the new North American datums (horizontal and vertical), the prediction of deflections of the vertical and geoid undulations, error propagation, inertial surveying, satellite systems, satellite long-term positioning and the automation of astronomic observations.

A Geodetic Survey long-term plan was completed to serve as a guide for the division's activities from 1980 to 1985.


In order to estimate the degree of compatibility in terms of longitude orientation between satellite Doppler and geodetic astronomical systems as realized by current observations, astrogeodetic (based on C10 pole, BII longitudes, and NWL90 satellite Doppler system) and gravimetric deflections of the vertical were compared at several hundred stations of the Canadian geodetic framework and U.S. triangulation networks. The geocentricity of the NWL90 system with respect to other systems such as the Goddard Earth Models and SAA Standard Earth is also analyzed by comparing satellite Doppler derived geoid undulations with GEN and SAA geoid undulations.


Une ligne de calibration gravimétrique a été établie, sous l'égide de la Commission Géodésique Suisse, entre Interlaken et le Jungfraujoch durant l'automne 1980 en accord avec les résolutions de l'Association Internationale de Géodésie. Cette ligne permet une calibration économique des gravimètres relatifs de terrain. Elle comprend sept stations, dont les deux extrêmes Interlaken et le Jungfraujoch (3500 m) ont une valeur de la pesanteur connue par des mesures absolues. La différence totale de pesanteur entre Interlaken et le Jungfraujoch vaut 604,7 µgal et l'analyse des résultats montre que la concordance des différences de g entre valeurs relatives et valeurs absolues est meilleure que 50 µgal (1 µgal = 10^-5 m/s²). Geo - HAWORTH, R.T. - "Interpretation of geophysical data in the northern Gulf of St. Lawrence and its relevance to lower paleozoic geology". Geological Society of America Bull, n° 80771, Vol. 89, 1091-1110, Aug. 1979.

The southern, subsurface boundary of the crustal section typical of the Grenville Orogen can be traced from the St. Lawrence River to western Newfoundland by means of a prominent gravity and magnetic gradient. North of the boundary, the gravity and magnetic data suggest that the upper surface of the Precambrian basement is irregular and is overlain by Paleozoic sedimentary rocks of the St. Lawrence Platform which outcrop in a regular undisturbed pattern on Île d'Anticosti. The areas known to have a thick Paleozoic section, such as Île d'Anticosti and offshore from Bay of Islands, Newfoundland, coincide with areas of relatively low gravity. East of Île d'Anticosti, however, an area with similar low gravity values and bounded by steep magnetic gradients was previously thought to be underlain by a thin Paleozoic section on the basis of seismic refraction data.
obtained from nearby areas. If the Paleozoic section is indeed thin, the
gravity and magnetic anomalies can be explained by the presence of an
anomalous intrusion with a minimum thickness of 4 km within the Precam-
brian basement. However, a more tenable interpretation, which is consis-
tent with the known sedimentary thickness elsewhere in the Central and
which does not contravene the refraction data, is that the Paleozoic section
thickens locally to at least 5 km. This thick section is postulated to
occur over a downfaulted portion of a sallment on the early Paleozoic mar-
gin of North America.

GEO - HAMORTH, R.T., LEFORT, J.P. - "Geophysical evidence for the extent of the

GEO - LEFORT, J.P., HAMORTH, R.T. - "Geophysical study of basement fractures on
the western european and eastern Canadian shields : transatlantic correla-


GEO - HAMORTH, R.T. - "Appalachian structural trends northeast of Newfoundland and
t heir trans-Atlantic correlations". Tectonophysics, n° 64, 11-130, 1980.

THP - LACHAPPELLE, G., SCHWARZ, K.P. - "Empirical determination of the gravity ano-
mal covarience function in montaneous areas". Survey and Mapping Branch, 126-127, 1979.

An evaluation of the empirical gravity anomaly covariance function using
over 95 000 surface gravity anomalies in the North American Western Cor-
dillera was carried out. A regression analysis of the data exhibits a
weakly quasi-linear correlation of free air gravity with heights. This
height correlation is removed from the free air anomalies prior to the nu-
merical evaluation of the gravity anomaly covariance function. This cova-
riance function agrees well with that evaluated previously by the authors
for the remainder of Canada. A possible use for such a covariance function
of "height independent" gravity anomalies in mountainous areas is descri-
bcd. First, the height independent gravity anomaly at a point of known
height is evaluated by least squares prediction using neighboring height
correlations calculated using linear regression parameters estimated
previously and added to the predicted height independent gravity anomaly
to obtain a predicted standard free air anomaly. This technique can be
used to demilitate the coverage of free air anomalies for subsequent use in
inertial formulas of geophysical geodesy, e.g., those of Stokes and Vening
Weijers. This method requires that point topographic heights be given on
a grid.

B.G - SCHWARZ, K.P., LACHAPPELLE, G. - "Local characteristics of the gravity anomaly

Using a data set of 260 000 gravity anomalies it is shown that common char-
acteristics for a local covariance function exist in an area as large as
Canada excluding the Rocky Mountains. After eliminating global features by
reforncing the data to the GEM-10 satellite solution, the shape of the co-
variance function is remarkably consistent from one sample area to the
next. The determination of the essential parameters and the fitting of the
covariance function are discussed in detail.

To test the reliability of the derived function, deflections of the ver-
tical are estimated at about 230 stations where astrogeodetic data are
available. Results show that the standard error obtained from the discon-
nectances is about 1" for each component and that the error covariance ma-
rix of least-squares collocation reflects this accuracy remarkably well.

NAG - HILES, P.R., SANDERS, M.R. - "Geophysical studies of the Gulf of Aden, Gulf

This report contains magnetic, gravity, seismic and bathymetric data col-
clected by the I.O.S. projects during Cruises 3/75 and 4/75 on R.R.S.
 Shackleton in April - May 1975. The profiles and track charts were all gen-
gerated automatically from data files set up from the shipboard data log-
gers. Included in the report is a description of methods used in data ac-
quision and preparation.

511 - ANGER, G. - "Uniquely determined mass distributions in inverse problems". (Presented at the 3rd International Symposium Geodesy and

We consider a special class of inverse problems. The right-hand side $v$
(sources of a linear differential equation) is determined from boundary
values or initial values. Such problems arise in geophysics, heat
conduction, diffusion processes, scattering theory of elasticity, wave
propagation and so on. In geophysics the gradient of the earth's potential is
known on the boundary of the earth. As a first step in these considera-
tions we assume all mass distributions inside the earth which produce
outside fields are known. This set is convex and weakly compact. The extre-
me mass distributions of this set are uniquely determined if the support
of the mass distribution is fixed. The other mass distributions are not
uniquely determined. In this paper we characterize extreme mass distribu-
tions for the Laplace equation. At first we develop euclidean ideas for the one-
dimensional Laplace equation without deep results of mathematics.

511 - DOBACZENIUS, K. - "The geoid problem in Nowadays works". (Presented at the 3rd International Symposium Geodesy and

In present day investigations of the Earth's figure, Earth's structure,
continential drifts, geodynamics, in determining absolute position in spa-
ce of terrestrial triangulation points a good geoid is necessary. In the
world the geoid has been determined from astrogeodetic and gravimetric data
for local territories. At present the geoid can also be determined from
Earth's satellites, Moon observations, altimetry, as well as space inves-
tigations. Even though the geoid is defined univocally, in practice sev-
eral kinds of geoids are distinguished. Depending on the kind of data used
for geoid determination various figures can be obtained between which con-
siderable differences in undulation can be observed.

Depending on the kind of observations and elaboration method the following
categories of geoids can be distinguished:
1. global or generalized geoid,
2. free-air geoid,
3. astrogeodetic geoid,
4. levelling geoid etc.

Corresponding to various solutions for each category there are a number of

The author shows that the inversion (0, a^2) gives a method to replace the Whittaker's problem exterior to a sphere (0, a) by the Dirichlet's problem interior to the same sphere. The computation of an harmonic potential inside a sphere (0, a) is then possible through a development in orthogonal harmonic polynomials for which some formulas have been given by WHITTAKER. The use of these formulas lead to different recursive formulations; some of these seem to be efficient in practice.


The geoid is considered as a certain equipotential surface of the earth gravity field. This paper is concerned with a specification of this equipotential surface. The final proposition for a definition is based on the geodetic height system by an inclusion of given information from oceanographic work on sea surface topography.

The realization of the geoid should be made in two steps. First, the existing height systems are corrected by information about sea surface topography. An equipotential surface is then chosen in such a manner that the sum of the squares of the deviations to the corrected height system is a minimum. This particular equipotential surface is called the geoid.

A least squares procedure is derived for a realization of this definition. The precise Cartesian coordinates of geophysical stations and in addition the quasiglobal undulations and the normal heights in these points are needed as data. The unknown corrections to the various height data influence, of course, the gravity anomalies and consequently the geoidal undulations. This influence is regarded in the least squares procedure.

Using the procedure described above a method is developed for the separation of the altimeter bias δA from the zero order term c_0 of the geoidal undulations.


As it is well known, the geoid is composed of two rotation symmetrical forms as their sum, the deep seated sources produce - corresponding to their global nature - rotation symmetrical forms. We succeeded to determine the two such forms of a general position suitable to approximate the geoidal figure in a most accurate manner. After removal of these values, the still remaining residual undulations contain purely the effects of surface inhomogeneities, these have their origin in initially balanced masses, partly they are of dynamical nature. Their amount is about one third of the undulations brought about by deep seated sources.


Two previously published computational methods in determining absolute orientation of geodetic datums are applied to the European Datum (1950). Using plumb line deflections, geoid heights and satellite data results for several types of combinations are presented. In comparison to classical least-squares approaches alternatives on applying collocation procedures are discussed and first numerical data are outlined.


Using a combination of satellite determined potential coefficients and free-air gravity anomalies a global (isotropic) covariance function for the anomalous potential of the Earth was computed by TSCHERNING and RAPP in 1974.

The variation (as expressed for example through this covariance function) is uniquely determined by mass density anomalies, but the inverse relationship is not unique.

Uniqueness may be imposed by requiring for example the mass density anomaly function to be harmonic (and correspondingly the anomalous potential harmonic inside the Earth). Under these and slightly different circumstances are cross covariances between density anomalies and derivatives of the anomalous potential derived. Corresponding to a free-air gravity anomaly variation of ±42 mgal (at the surface of the Earth) the mass density variation becomes ±0.80g/cm³ at the surface of the Earth and ±0.04 g/cm³ in 10 km depth.


Based on the method of virial equations, the integro-differential equations governing the physical figure of the earth are derived. Volume forces of gravitational type and surface forces of stress type are assumed. Initial stress is represented by the H. TAKEUCHI (1951) model. Equilibrium figures for different earth models - elastic, elastic under initial stress, liquid - are studied. It is proved that non-liquid earth models allow ellipsoidal structures of Haclaurin type.


Des mesures sur le terrain des anomalies de la pesanteur ont permis d'édifier des cartes gravimétriques. Les auteurs nous présentent quelques interprétations de certaines de ces anomalies qui permettent une meilleure connaissance du sous-sol.
T. KRÄMER proposed the use of collocation with kernel functions for the approximation of a potential function on the earth surface as well as in local regions of a sphere. Starting from the smoothing criterion of the least norm of the horizontal gradients on a sphere, a neighbourhood criterion was derived taking into account smoothness as well as stability properties of the series evaluation. It is finally shown how to choose the kernel functions in order to obtain a smooth interpolation function at the surface of the earth.

The least squares estimator is derived for a random stochastic process implied by one or two heterogeneous random stochastic processes on a sphere. The solution can be regarded as least square collocation in the continuous case. When the method is applied in physical geodesy the statistical expectation is usually substituted by the global average and the method will then give the minimum mean squares errors of the estimated quantities. The solutions can also be considered as generalisations of the classical integral formulas in physical geodesy for heterogeneous data information.

Based on the method of virial equations, the integro-differential equations governing the physical figure of the earth are derived. Volume forces of gravitational type and surface forces of stress type are assumed. Initial stress is represented by the H. TAKEUCHI (1951) model. Equilibrium figures for different earth models (e.g. elastic under initial stress, liquid) are studied. It is proved that non liquid earth models allow ellipsoidal structures of MAÇLARIN type.

In a least squares adjustment (a minimum variance solution) using the technique of variation of coordinates (observation equations), a key result is the co-variance (dispersion) matrix of parameters. Assuming that standard errors of observations are used in the formation of the normal equations, rather than relative weights, this dispersion matrix gives the estimates of standard errors for the parameters solved for in the adjustment. A method will be presented which allows the designer of the observing plan to alter this dispersion matrix, which may not meet user requirements, so that it will meet user requirements and, from its inverse, solve mathematically for the selection and quality (accuracy) of the observations required to form the altered dispersion matrix of parameters.

In contrast to tracking station fitting procedures where non-uniformly distributed data are applied to the detailed distribution of the relative locations of origins of specific coordinate systems (like GEM, SE etc.) uniformly distributed mean geoid values are used in an alternative approach. Those geoid data are obtained from sets of truncated spherical harmonic coefficients related to specific systems (like GEM 8, SE 14, GEM 2 etc). It is shown that relatively small (in general less than 1 m) distances between the origin of GEM 8 and those of different other systems are found for recent models. The study corroborates the results obtained mainly in the US and reveals clear disagreement with previous investigations where separations between origins of the order of > 10 m were detected.

In the paper only medium and short wave length effects affecting the measured relative velocity have been considered. The influence of measuring errors is discussed and it is shown that only satellite-to-satellite observations with accuracy better than 0.1 mm/sec will give an improvement of the geoid. Finally, some results on the combination of low-low satellite-to-satellite tracking and terrestrial gravity data are given.

The possibility of improving the convergence of Molodensky's is considered. The new formulas are derived for the solution of the geodetic boundary value problem. One of them presents an expansion condition which involves a linear combination of Stokes' constants and surface gravity anomalies. This differs from the usually used relation by the appearance of additional terms dependent on second order terms with respect to the elevations of the earth's surface. The formulas are derived...
The satellites of the Global Positioning System (GPS) offer an important new geodetic resource making possible a highly accurate portable radio geodetic system. A concept called SERIES (Satellite Emission Radio Interferometric Earth Surveying) makes use of GPS radio transmissions without any satellite modifications. By employing the technique of very long baseline interferometry (VLBI) and its calibration methods, 0.5 to 3 cm three dimensional baseline accuracy can be achieved over distances of 2 to 200 km respectively, with only 2 hours of on-site data acquisition. The use of quasar referenced ARIES sites and VLBI to establish a sparse fundamental control grid will provide a basis for making SERIES GPS measurements traceable to the time-invariant quasar directions. Using four SERIES stations deployed at previously established ARIES sites, the GPS satellite apparent positions to be determined. These apparent positions then serve as calibrations for other SERIES stations at unknown locations to determine their positions in a manner traceable to the quasars. Because this proposed radio interferometric configuration accomplishes its signal detection by cross-correlation, there is no dependence upon knowledge of the GPS transmitted waveforms which might be encrypted. Since GPS radio signal strengths are 10^12 stronger than quasar signals, a great reduction in telecommunication sophistication is possible which will result in an order of magnitude less cost for a SERIES GPS station compared to a quasar based mobile VLBI system. The virtually all-weather capability of SERIES offers cost-effective geodetic monitoring with applications to crustal dynamics and earthquake research.

A system of miniature radio interferometer terminals is proposed for the measurement of vector baselines with uncertainties ranging from the millimeter to the centimeter level for baseline lengths ranging, respectively, from a few to a few hundred kilometers. Each terminal would have no moving parts, could be packaged in a volume of less than 0.1 m^3, and could operate in all weather conditions. These units would receive radio signals from low-power (< 1 W transmitting) Earth-orbiting satellites. The baselines between units could be determined virtually instantaneously and monitored continuously as long as at least four satellites were visible simultaneously. Acquisition of the satellite signals by each terminal would require about one minute, but less than a second of signal integration, and the collection of only a few kilobits of data from two receiving units would suffice to determine a baseline. Different baseline lengths, weather conditions, and desired accuracies would, in general, dictate different integration times.

Formulas for computing geodesics on the bi-axial ellipsoid through Gaussian quadrature are shown, the estimation of computational errors, truncation and roundoff errors, for the quadrature is carried out in terms of examples found in [3] together with those which consist of near anti-podal points on the neighborhood of the equator, are computed with the evaluation of the computational errors.
A recurrence relation has been derived to obtain the derivatives required for the Taylor expansion of the theoretical gravity formula in powers of latitude. The computations using the relationship derived can provide easily all required derivatives for any reference ellipsoid.

Modern high accuracy measurements of the non-rigid earth are to be referred to four-dimensional, i.e., time- and space- dependent reference frames. Geodynamic phenomena derived from these measurements are to be described in a terrestrial reference frame in which both space- and time- like variations can be monitored. The ultimate goal of this study is the establishment of a reference frame, moving with the earth in some average sense, in which the geometric and dynamic behavior of the earth can be monitored, and whose motion with respect to inertial space can also be determined.

The study is conducted in several parts. In the first part problems related to reference frames are investigated, while subsequent parts deal with positions, i.e., with reference origins and scale. Only the first part is treated in this paper.

A key element in any attempt to achieve a 0.1 m precision in geoid representation via satellite altimetry is obtaining an ephemeris of comparable precision, or circumventing this requirement by adjusting the ephemeris in some way, together with the geoid. The first possibility requires extensive satellite tracking and involves an enormous number of adjustable parameters in the long arc approach.

The second possibility allows for a piece-wise treatment of short orbital arcs considered mutually independent, in which slight adjustments of the state vector parameters can compensate for an inherent modeling error. The main question to be answered when pondering the possibility of using the short arc adjustment in SEASAT-A altimetry reductions is whether or not this method is inherently capable of representing the detailed geoid with 0.1 m precision. An analysis of computer simulations provides at least a partial answer to this question by pointing out the necessary conditions in order to achieve the desired precision; under certain favorable circumstances, not described in detail, these conditions could prove to be almost sufficient.

It is shown that for linear parametric adjustment models all the least-squares equations can be obtained from a commutative table of observations and parameter spaces are regarded as covariant. Their contravarient counterparts are derived through the metric property of the covariant matrix of the observations.

The method of Bjerhammar is studied in the continuous case for a sphere. By varying the kernel function, different types of unknowns ($u^2$) are obtained at the internal sphere (the Bjerhammar sphere). It is shown that a necessary condition for the existence of $u^2$ is that the degree variances ($\sigma_u^2$) of the observations are of an order less than $n^{-1}$. According to Kaufman's rule this condition is not satisfied for the earth's gravity anomaly field ($\sigma_u^2 = n^{-1}$) but well for the geopotential ($\sigma_u^2 = n^{-1}$).

An examination is made of the error of alignment of bars (tapes, wires) in the rectilinear measurement of distance between two points in space. This is evidently a systematic error, of which one derives the frequency function. Starting from the two dimensional case for which we obtain a general formula for the moments, it is possible to determine the frequency function in the three-dimensional case being examined, and a closed formula for its moments.

The Hookasalo term is discussed and the author arrives at the conclusion that its application to gravimetric observations is harmful and should be avoided.

The boundary value problem of physical geodesy has been solved with the use of a harmonic reduction down to an internal sphere using a discrete procedure. (For gravity cf. Bjerhammar 1964 and for the potential cf. Bjerhammar 1968). This was a finite-dimensional approach mostly with one-to-one correspondence between observations and unknowns on the sphere.

Earlier studies were made with the use of surface elements (on the sphere) with constant gravity. Integration over the surface elements was replaced by a discrete approach with the use of the distance to a point in the centre of the surface element. See Bjerhammar (1968) and (1969). This approach was later presented as a "reflexive prediction" technique for a weakly stationary stochastic process. Bjerhammar (1974, 1976). Krarup (1969) minimized the $L^2$-norm of the potential on the internal sphere. It will here be proved that the two solutions are identical for a proper choice of the radii of the internal spheres. The proof is given for a spherical earth with selected choice of "carrier points".

The convergence problem is discussed. The $L^2$-norm solution is found convergent for the fully harmonic case. Uniform convergence is obtained in the non-harmonic case with the used of the original procedure applied in accordance with the theorems of Keldysh-Lavrentiev and Yamabe.
The geopotential on and outside the earth is represented as a series in surface harmonics. The principal terms in it correspond to the solid harmonics of the external potential expansion with the coefficients being Stokes' constants $C_{\text{MM}}$ and $S_{\text{MM}}$. The additional terms which occur near the center due to its non-sphericity and topography are expressed in terms of Stokes' constants too. This allows performing downward continuation of the potential derived from satellite observations. In the boundary condition which correlates Stokes' constants and the surface gravity anomalies there occur additional terms due to the earth's non-sphericity and topography. They are expressed in terms of Stokes' constants as well. This improved boundary condition can be used for upward and downward continuation of gravity anomalies. Simple expressions for the error terms are derived as explicit functions of the surface anomalies and its derivatives. The formula for the disturbing potential on the surface is derived in terms of the surface anomalies. All the formulas do not involve the earth's surface inclinations.

A minimum fill-in scheme, published by Yale University and not previously discussed in the geodetic literature, has been tested on four typical triangulation networks. Its computer storage requirement has been compared with that of the profile method using Soay's station reordering algorithm. Tests have been carried out for terrestrial and mixed terrestrial/satellite networks.

The conclusion is that the fill-in scheme is a viable method for geodetic networks, and in some cases, it needs less computer storage than the profile method.

The analysis of deformations and reductions of the geodetic networks in general gravity-invariant and potential-invariant representations of the actual gravity field of the Earth by normal (theoretical) gravity field has been presented.

It has been shown that the linearized formulae of the scale factor and linearized formulae of the transformed azimuth and zenith distance induce the characteristic of transformation of three-dimensional networks in inozedhal-type of gravity-invariant representation. In inozedhal-type of gravity-invariant and potential-invariant representations the two-dimensional (horizontal) networks are transformed conformally. In isomorphetic cases of gravity-invariant and potential-invariant representations, the two-dimensional networks are transformed equivalently with respect to the scale and angles, but non-conformally.

The error in the mean earth ellipsoid based on the basis of Doppler or laser observations of artificial earth satellites or radar altimeter observations of the ocean surface from a satellite depends upon instrument precision, on uncertainties in the specification of the earth's gravity field at both long and short wave lengths, on uncertainties in the origin of the coordinate system, on modeling errors in ionospheric (except laser) and tropospheric refraction, and, for altimetry, on oceanographic effects. The magnitude of the uncertainty in the computed ellipsoid will vary depending on the size of these errors and on the number and distribution of observations. Review of the data used in various analyses indicates that differences in the computed ellipsoids are consistent with those expected due to the various error sources and that the best fitting ellipsoid has a semi-major axis of 6378165 ± 2 m.

The error caused by using the Honkasaalo tidal gravity term in geoid computations with Stokes' formula is discussed. It is pointed out that the relationship of the errors to interaction between the Honkasaalo term and the weight function in Stokes' formula.

A numerically efficient solution strategy is developed for the second-order design of a free distance network. It is based on the fact that the direct (non-canonical) way has a regular least-squares solution. A flow chart illustrates the direct construction of the final equations, using a list of point coordinates and a list of projected distances as input. Results are shown for an actual network (11 points, 47 distances), considering different types of criterion matrices: Taylor-Karman structure with homogeneous isotropic point error ellipses and special Taylor-Karman structure with heterogeneous anisotropic relative error ellipses. Finally, the construction of a criterion matrix $Q_\lambda$ is briefly discussed in the case that a certain number of quantities shall be derived from the network.

The determination of high frequency variations in Earth's rotation from doppler satellite observations. The determination of high frequency variations in UT-1 and a component of pole position from a single pass of Doppler observations of a Navy Navigation Satellite is affected by instrument errors and uncertainties in the gravity field and atmospheric drag forces used in computing the satellite orbit. For elevation angles above 20°, instrument errors contribute about 2 m/sec to the determination of UT-1 and "0.3 to the determination of pole position. Gravity and drag errors contribute about 0.03 of correlated errors. Net gravity errors were transferred by statistical analysis of residuals after fringe errors and reduced by drag-compensating devices aboard future Navy Navigation Satellites. Since 20 Doppler stations nominally acquire about 100 passes each day, daily observations of UT-1 and pole position could achieve precisions of 0.2 m/sec and "0.05, respectively, assuming half the passes contribute of Doppler results for two day solutions is about 50 cm for pole position and 1 m/sec for high frequency variations in UT-1.

The development of relatively inexpensive satellite receivers in the early 1970's has resulted in cost-effective applications of satellites for a variety of geodetic surveying needs. Currently achievable accuracies range from 10 to 20 centimeters. The RAVESTAR Global Positioning System, now
under development by the Department of Defense, incorporates advanced technology which has the potential capability of revolutionizing satellite geodesy.

Several concepts for utilizing GPS signals are briefly reviewed, and another concept, called the reconstructed carrier phase method, is described in some detail. This concept is being pursued by the Defense Mapping Agency, National Oceanic and Atmospheric Administration, and the U.S. Geodetic Survey. These agencies have numerous requirements for accurate position- tioning. Several prototype receivers are planned to be available for testing in mid-1982. These receivers should be highly portable, consume little power, and obtain base line accuracies of several centimeters in several hours of observation time. However, water vapor radiometers will be needed in order to achieve the full accuracy. Initial simulation results utilizing the reconstructed carrier phase method are included.


Geodetic positioning accuracies obtained from range, integrated Doppler and double difference interferometric phase observations from a constellation of twenty-four Global Positioning System satellites are presented. It is demonstrated that GPS range and Doppler observations will provide sufficient accuracy for the estimation of geodetic coordinates. However, the instability of the receiver atomic oscillator will limit the usef ulness of these observations in providing rapid first-order baseline determination. Interferometric phase measurements twice differenced to eliminate clock error appear as an alternate procedure for providing such accuracies.

524. - RUOČIKI, G. - "The influence of structure of observed objects and construction of measurement devices on the thermal imagery". IGiK, tom XXVII, ZESZYTY 7 (65), 3-68, 1980.


GRW - "Information astronomical union, Union astronomique internationale".

SYT - "Laser tracking quick-look data catalog and earthpole positions short merit campaign".

This is a catalog of quick-look laser data received and processed by the Smithsonian Astrophysical Observatory (SAO). It is intended to serve as a guide to the final data that will ultimately become available and as an indication of the results of the laser tracking efforts known as the short MERIT (Measurement of Earth Rotation and Intercomparison of Techniques) campaign from August through October 1980.


The following quantities are considered: geoidal undulations N, gravity anomalies A, deflections of the vertical Δν, a fictitious surface density ρ, the vertical gradient of gravity anomalies Δδ. These quantities are interrelated by linear orators having the spherical harmonics as eigen-functions. If the covariance of one of these quantities is specified, that of the others can be computed. Attention is focused on techniques that have been worked out for the determination of the covariance of one of the residual quantities and the covariance of the other quantities tends to have certain theoretical properties. These are a predetermined number of zeros as well as negative correlation at certain predetermined distances.


The principle of the Väisälä interference comparator and a description of the measuring method are presented. Some remarks on the method are made.


Measurements of geodetic base lines and calibrations of 24 m invar wires applying the Väisälä multiplication method are based on 1 m length standards. Consequences for the measurement of these standards result from their geometrical form. A review of calibrations between 1933 and 1970 is given and the results are discussed. The methods used for calibrations differ in principle with different sources of systematic errors. Against the uncertainty of the multiplication, e.g. 6.10⁻⁸ of 864 m, the coincidence of the calibrations deviating one from the other be more than 10⁻⁷ m should be improved. For this reason, it is proposed to introduce standards with plane and mutually parallel surfaces.

The reasons for the direct measurement of the Humala comparison baselines with the Väisälä comparator are set out. The main data of the nine Humala measurements are given. After description of the administrative arrangements leading to the standard baselines abroad, the results of measurements of these baselines are given. On the basis of mutual comparisons and of absolute length determinations the lengths and annual changes of the Väisälä quartz meters are computed. The provisional results of all 19 measurements, published or unpublished, are reduced according to the above lengths in one consistent system.


Instruments of the Bureau International des Poids et Mesures (BIPM) for the base of geodetic length measurement and a 24 m dialmeter are described. Several measures for stabilising invar and investigations are carried out by the BIPM for ensure good reproduction and great accuracy in the length of invar wires and tapes. Examples give some idea of the temporal stability of different geodetic gauges.

Several one-metre bars of fused silica with spherical end surfaces are measured at the BIPM using different methods. The obtained accuracy corresponds to expectations.


The paper deals with the results of thermal expansion investigations of invar wires. The expansion measurements were performed at the temperature range -10°C to 50°C. The experimental curves were approximated using the third order polynomial. It was found that the shape of the expansion curves depends on the load.


Baseline measurements have a long tradition in Finland. The first baseline was measured as early as 1735 by the French astronomer Pierre de Laquet. Three baselines were measured for the Russian-Scandinavian Arc Measurement the first of them in Elimäki in 1844, the second in Olvi in 1865, and the third in Ylitornio in 1951. Russian topographers measured the baselines to Aland, Kemiö and Sulva with invar tapes in the years between 1908 and 1912. In addition, one baseline was measured to Reniäjärvi by the National Board of Survey in 1920. None of the above baselines are in use any more. Part of them have been destroyed, the rest do not fulfill the accuracy requirements of modern geodetic networks.


A survey of the development of the high precision length measurements at Deutsches Geodätisches Forschungsinstitut since 1954 is given. After a brief historical review information of the following projects of investigation are presented:

2. Determination of the length of the standard base line Munich and the 60 m comparison base line by the Väisälä light interference comparator.
3. Base lines and base extension nets.
4. Measurements and computations for the European satellite trajectories.
5. The system of quartzimeters at the BGI.


Invar metal was discovered at the beginning of the century by BÉHOT and GUILLAUME, directors of the Bureau International des Poids et Mesures. Invar has been used in the form of wire, and sometimes tape (USA), for measuring the geodetic base required for scaling of triangulations over large countries. When invar was first employed, however, certain difficulties were encountered due to ageing, the temperatures to which it was exposed and way in which it was handled.

During the second half of the 20th century, invar's importance in geodesy decreased as distanceimeters became increasingly reliable and precise. Invar has therefore only two specific areas where it still has a significant part to play. In the short term, it will continue to be used for measuring the bases needed for calibrating distanceimeters. In the longer term, a standpoint, it is still the one and only solution for the geodetic approach dealing with large engineering structures in the 100 m range.

Over a 20-year period, more than 30 km of invar wire were used for positioning the magnet components of the large accelerators of the European Organization for Nuclear Research (CERN). The Survey Group had to overcome invar's capricious behaviour, develop new instruments and use methods which would be free from its shortcomings.

The present paper is essentially concerned with the description and historical background of this unique experience. From an intensive study of results a limit is obtained for the accuracy of invar wire measurements in conditions similar to those of metrological laboratories.


This paper deals with cases of distance measurement in which temperature affects the accuracy significantly : measurements with invar tape, EDM, and interferometer. An invar tape suspended near the ground is heated by radiation from sun, sky and surface. Its temperature is generally higher than air temperature, often by several degrees, so direct measurement of the tape temperature is recommended. Suitable instruments using contact sensors are now available. The problem of representing the temperature of an EDM line may eventually be solved by using two-wavelength instruments, in the
meantime, improved atmospheric models are available. The Heat Balance Turbulent Transfer Model, which has shown promising results in field tests, is outlined. Modern interferometers are capable of high precision over shorter distances. The accuracy and the range are affected by temperature fluctuations in the atmosphere. These are minimized by observing when the sky is heavily overcast, or close to the times of sunrise and sunset. Measurements should not be made under conditions of calm or very light wind.


The possibilities of using the numerous distance measurements in the densification network for a general network scale determination are surveyed.


In a part of the Swiss first order triangulation net electronic distance measurements have been carried out in the years 1969-1977 by means of Geodimeter 8.

The comparison of the measured distances with the corresponding side lengths according to the "official" coordinates show regional scale differences. The recently new adjusted triangulation net fits distinctly better with the measured distances, except one point whose trigonometric configuration is ill-conditioned.

Although the block CH (Switzerland) of the European Triangulation Network (ETN/C) does not cover the whole national network and the distances measured before 1973 were already introduced in the adjusted ETN/C II "1973" of this block, the measurements of 1974-1977 were compared with the resulting distances and show good agreement.


The method of measuring lines in groups was applied in a survey of seven radial lines ranging in length from 32 to 55 km. Measurements made a month apart and adjusted to a common scale were totally consistent and agreed within 0.2 parts per million. These remarkable results encourage further studies in the application of this method.


The extensions to the Väisälä base line in South Africa were measured using a high resolution Electromagnetic Distance Measurement (EDM) as a comparator. The results show that the accuracy of the extension is of the order of 3 parts in 10^7 which is considerably better than the absolute accuracy that can be obtained with the instrument.


The basic formula of calibration and determination of cyclic errors are given for electromagnetic distance meters. For the direct solution filter equation with practical examples are proposed.

For the deducing of high precision distances the measuring method in all combinations are given.

For the determination of the scale factor the problems of comparative baselines outlined.

The method can be used for the deducing of high precision distances being an everyday task in movement and deformations studies.


Between two pillars 50 m apart is an U-shaped bar where the reflector of an interferometer can be moved with an electric motor.

A standard steel tape is measured with the interferometer. The standard tape can then be photographed together with another tape at every tenth meter and the differences between the graduations measured in a special apparatus.

A reflector can be automatically moved for certain intervals (2.5, 1.25 and 0.1 m) between 2.2 and 0.2 m to determine the calibration constants for an EDM-instrument.


We determined the time dependent trend and the resulting standard error in the length standards of various EDM-instruments. We observed two microwave instruments (MDA 4, CA 1000) and three light wave instruments (Geodimeter 8, MA 100, ME 3000) for about six years. The results obtained are valid for course only for the instruments investigated, but they should be characteristic also of the corresponding types.

To eliminate the error influence of a possible instability of the fine measuring frequency, the modulation frequency has to be measured in the field during a distance measurement. Such a procedure is reasonable at least for the Mekometro, because its frequency errors can exceed the errors in the representative refractive index.

At the Geodetic Institute of Karlsruhe we are therefore developing a small portable frequency calibration unit suitable for all EDM instruments.
The Hekometer was calibrated using the Numella Standard Baseline. Several distances from 24 m to 864 m were measured. All measurements made with the Hekometer are compared with known lengths. The concrete pillars of the interference baselines were used in the calibration. The stability of the pillars is studied and the projections of the Hekometer measurements onto the Standard Baseline treated in detail. The air temperature and pressure were carefully recorded at the Hekometer station and between the pillars observed. The observation method and the results are presented.

The zero and scale corrections are computed from the residuals i.e. the Hekometer measurement minus known distance. The temperature dependence of the scale correction is determined in the range 17-55°C. The distance dependence of the zero correction is also studied. The measuring ranges 10-50 m and 500-3000 m are studied separately.

Indoor calibration of Hekometer was carried out on two calibration lines. The maximum lengths of these lines are 75 and 280 meters and the lines consist of concrete pillars mounted on the ground rock.

Standard deviations of adjusted zero corrections were between 0.06 and 0.10 mm. It seemed obvious that the value of zero correction for range 1 depends slightly on distance in the case the distance is less than 100 meters.

It was also shown by simulation that certain deviations in the operation of the modulating cavity cause errors of the same type especially on short and very long distances.

If the Hekometer has the periodical phase error, it is probably very small. Therefore the test method used to determine this kind of error must be extremely accurate. The required accuracy of the measuring method was determined to be 0.02...0.03 mm, so that the error greater than 0.1 mm can be found out reliably. This requirement could be fulfilled by using the HP-laser interferometer and by applying Abbe's comparator principle.

The results from the two different half wavelengths were rather similar and the periodicity is obvious. However, the error is very small, about 0.05...0.10 mm. When an extremely high accuracy is desired, it is profitable to reduce the observations with corrections corresponding to the periodical error.

The National Geodetic Survey of the National Ocean Survey and its predecessor, the U. S. Coast and Geodetic Survey, have performed over 20,000 unique length determinations - distances varying from 6 km to 93 km. Thousands of line lengths have also been measured in the short to medium ranges (up to 6 km). Various techniques and instrumentation have been employed. Many of the individual lines have been observed using several different methods. Approximately 30 lines along the High Precision Transcontinental Traverse have been measured both by mechanical (steel tapes, invar tapes, etc.) and electronic means. A critical evaluation of these observations leads the author to pose the question: "Are accuracy estimates exceeding one part per million realistic?"


A report is given on the results of analyses of 1332 days of registration by an Ankonia GS 15 gravimeter. The fully corrected results of different analysis methods are in a good agreement. The effect of the resonance of liquid outer core is compared with the results of other European stations. Analysing intervals of the whole data set the variation of the resulting tidal parameters in time seems to be a hindrance for a further arising of the accuracy as well as for the universal validity of the results obtained by temporary measurements at a given station.

Pucin, A.H. - "Data compression filters coordinated with the least squares forms method /LSFM/ for the analysis of a polyharmonic signal".

The article describes a method of data compression filter construction for a polyharmonic signal. It is advisable to use such filters when processing large volumes of geophysical information. Polyharmonic signal amplitude dispersions being dependent on coefficients of data compression discrete filtering, it is possible to define the efficiency of the existing filters such as those of Venedikov and Lecolazet to analyze Earth tides. Data compression filter can be constructed as an optimal one with respect to perturbation spectrum.

Joo, T., and al. - "Recent vertical crustal movements of the Carpatho-Balkan region".

Mei - "MAKES TERRESTRES".


Description des principales Etapes de la mise au point de capteurs capa
mifs adaptés aux pendules horizontaux Verhaender-Melchior, by M. Van
Ruychekre.

Utilisation de capteurs capacitifs adaptés aux extensomètres, by M. Van
Ruychekre.

Sur la possibilité qu'ont les hétérogénéités horizontales du manteau
apparaître dans les observations de marées terrestres, by S.M.
Molodoshyn, H.V. ERANNER.

Influence des forces d'inertie sur les variations de marées de la force
de pesanteur et dos inclinaisons, by H.M. PARIKSKII, B.P. PETROV.

Calcul des nombres de charge pour le modèle terrestre 508 de
Gilbert et Deiwonski, by B.P. PETROV, M.V. Ivanova.

Koe, H., "Numerical investigations on the problem of Molodensky".

Ge - Kähle, H.G. - "Aktuelle problem der Geodäsie und Geodynamik".

W. A. Müller and H.-G. Köhler, "Novel methods for determining the crustal structure of the Alps in terms of seismic compressional wave-velocities and depths to the crust-mantle boundary in Switzerland and the Geophysical interpretation."
Tectonophysica, 71, 125-152, 1981.

In this paper a review is presented on the most recent results of geode-
tically determined recent crustal movements in Switzerland. These data are compared with the new Bouguer gravity map of Switzerland, and an attempt is made to discuss the geodetic and gravity results in connection with the crustal structure of the Swiss Alps in terms of seismic compressional wave-velocities and depths to the crust-mantle boundary.